



<http://algs4.cs.princeton.edu>

## 2.3 PARTITIONING DEMOS

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- *Sedgwick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*



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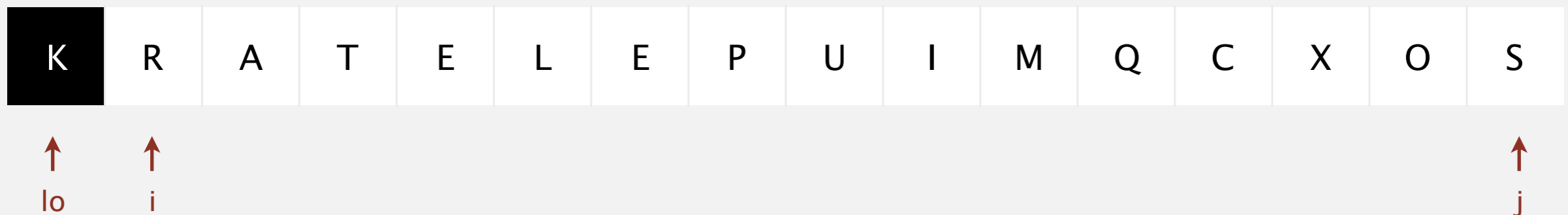
- *Sedgwick 2-way partitioning*
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- *dual-pivot partitioning*

# Quicksort partitioning demo

---

Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .



**stop  $i$  scan because  $a[i] \geq a[lo]$**



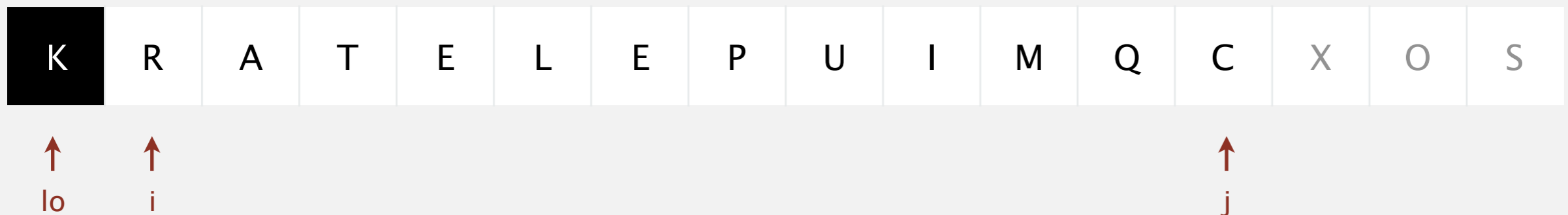


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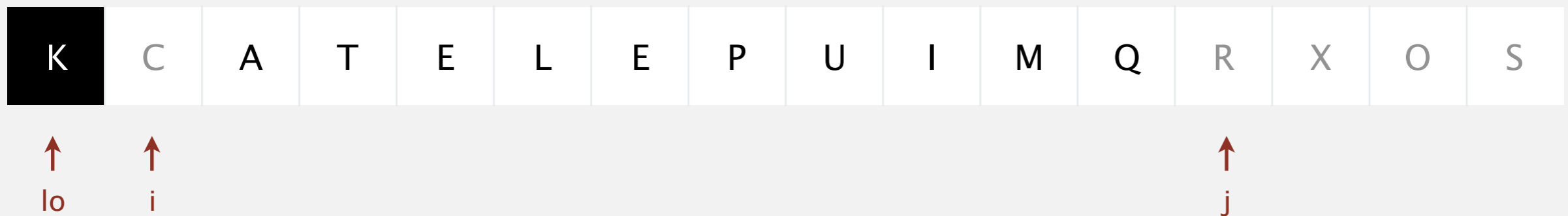
**stop  $j$  scan and exchange  $a[i]$  with  $a[j]$**

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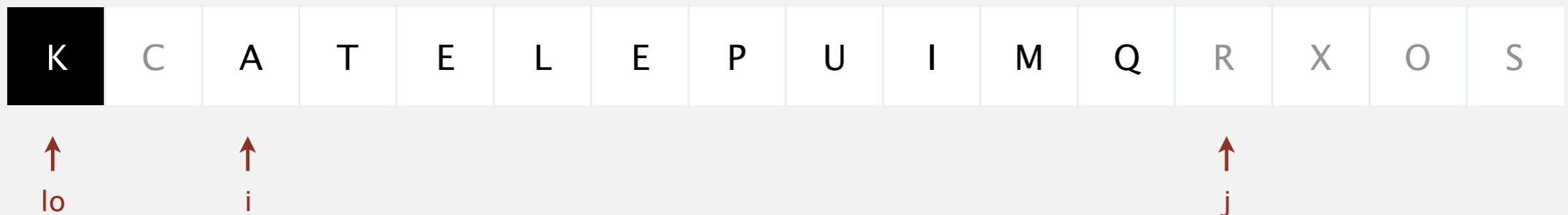


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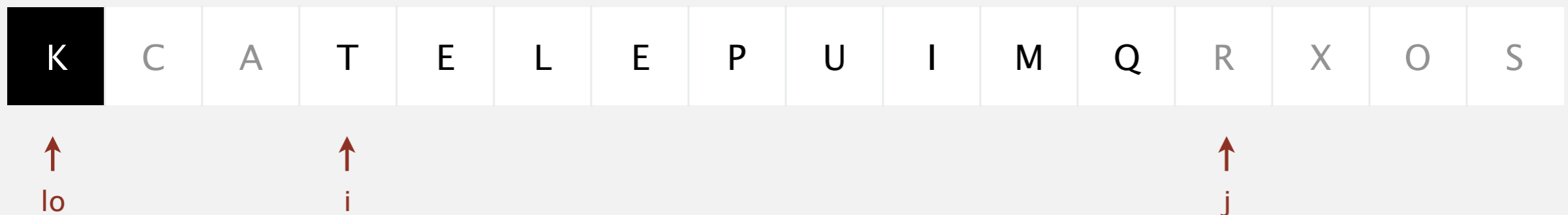


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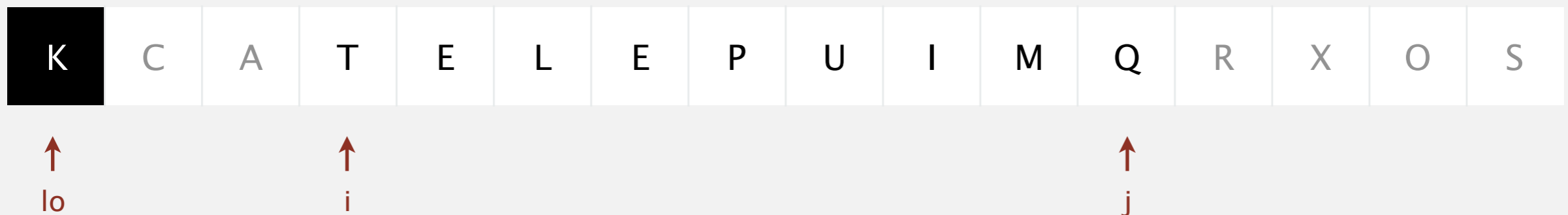
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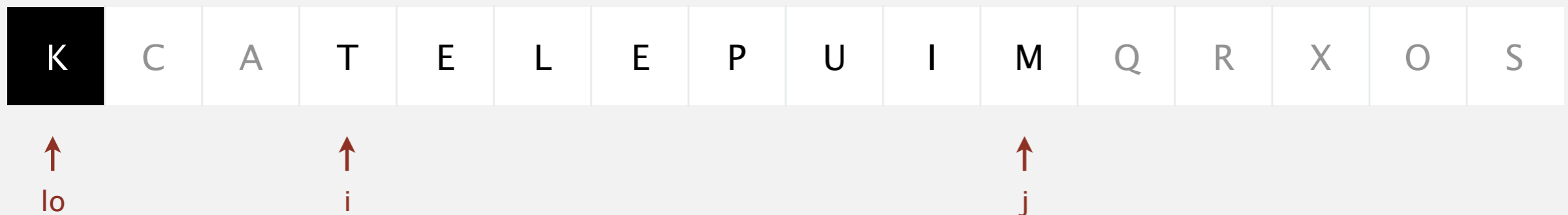


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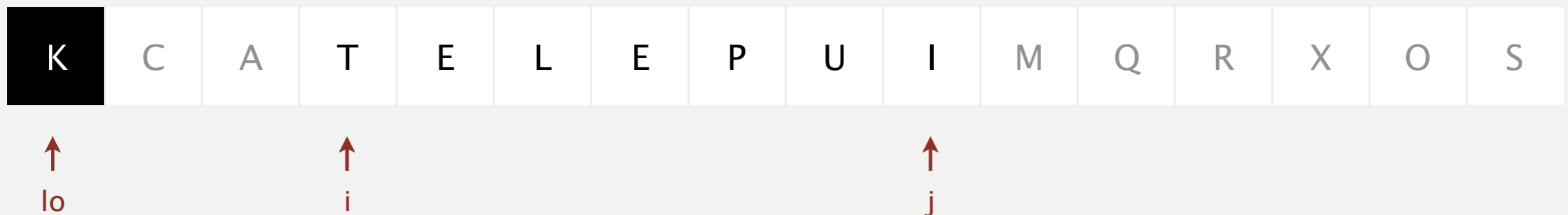


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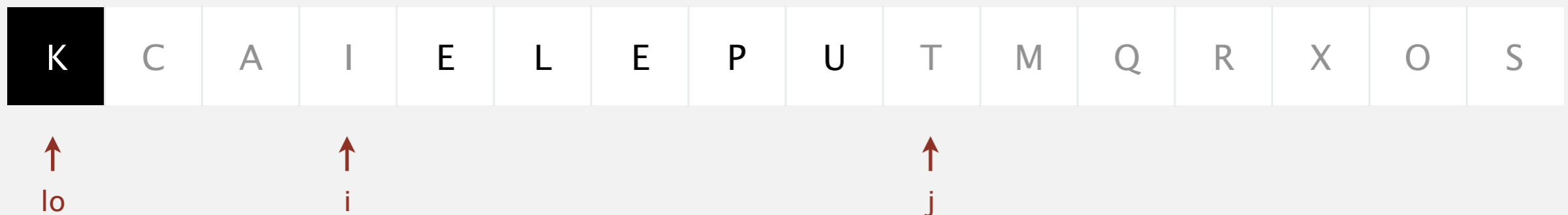
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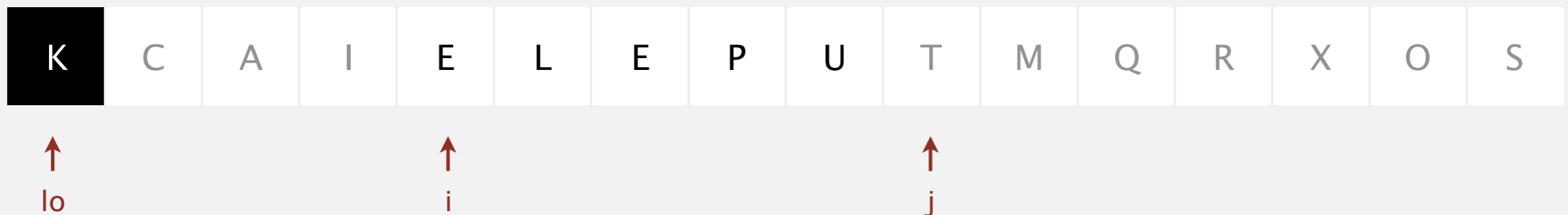


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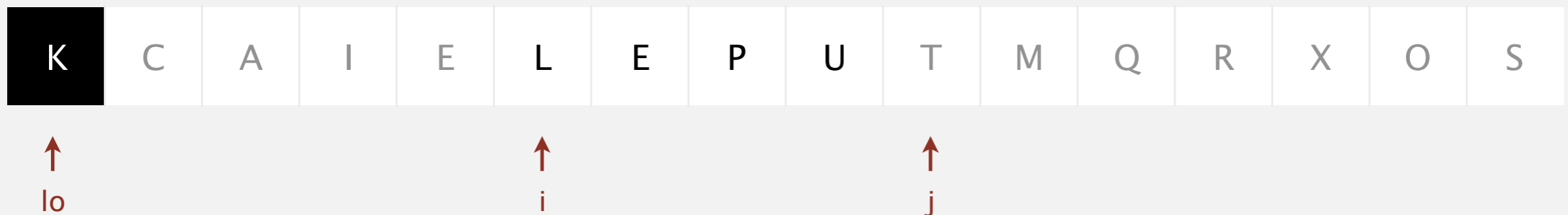


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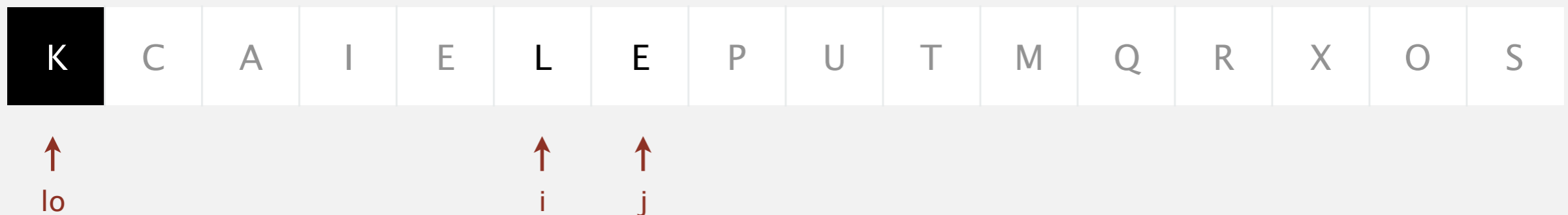


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**stop  $j$  scan and exchange  $a[i]$  with  $a[j]$**

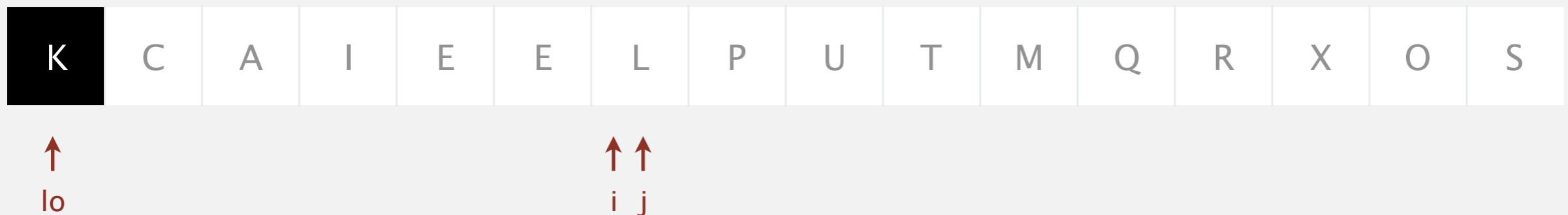


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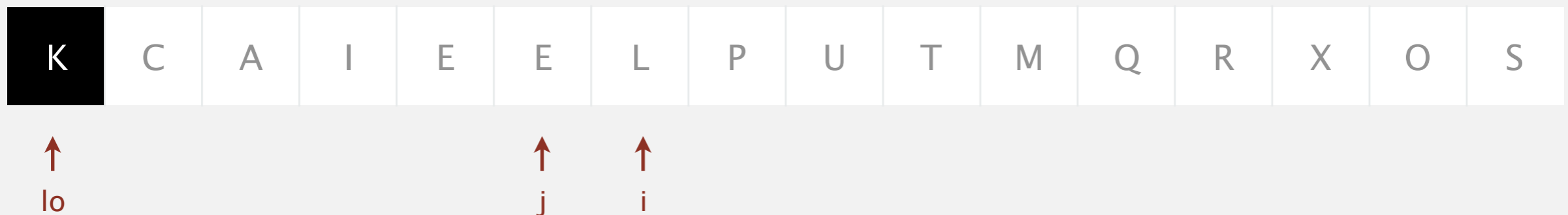
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**stop j scan because  $a[j] \leq a[lo]$**

# Quicksort partitioning demo

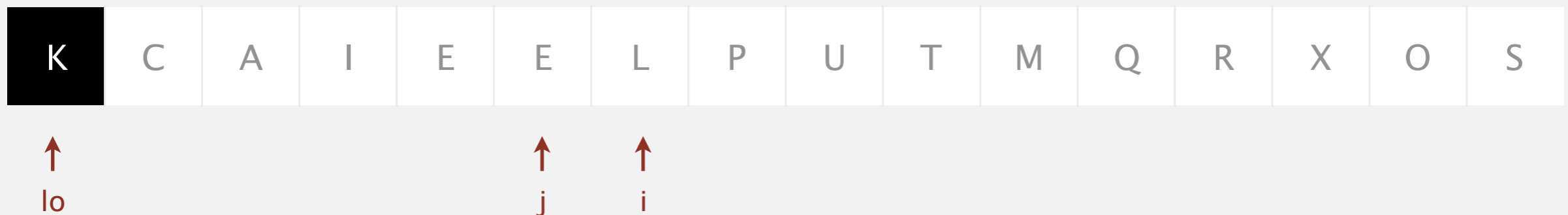
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- Exchange  $a[i]$  with  $a[j]$ .

When pointers cross.

- Exchange  $a[lo]$  with  $a[j]$ .



**pointers cross: exchange  $a[lo]$  with  $a[j]$**

# Quicksort partitioning demo

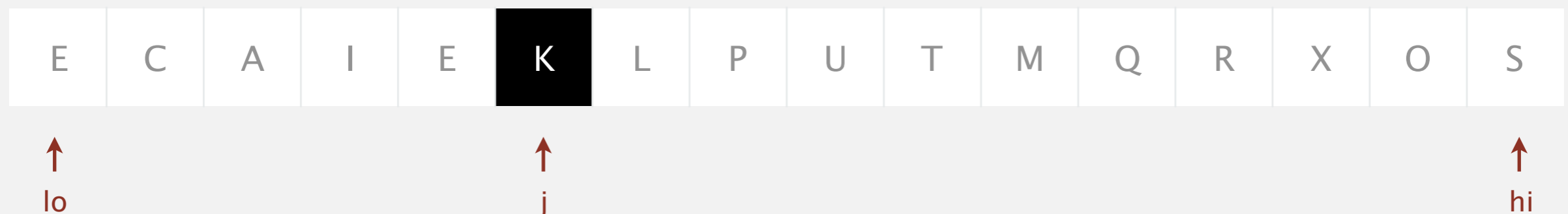
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**partitioned!**



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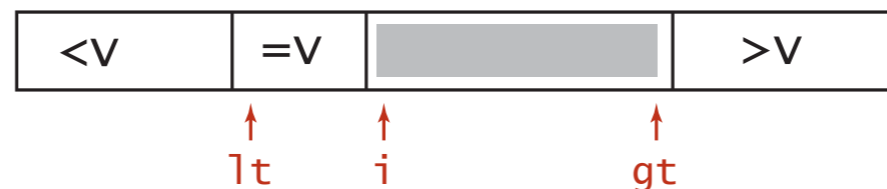
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



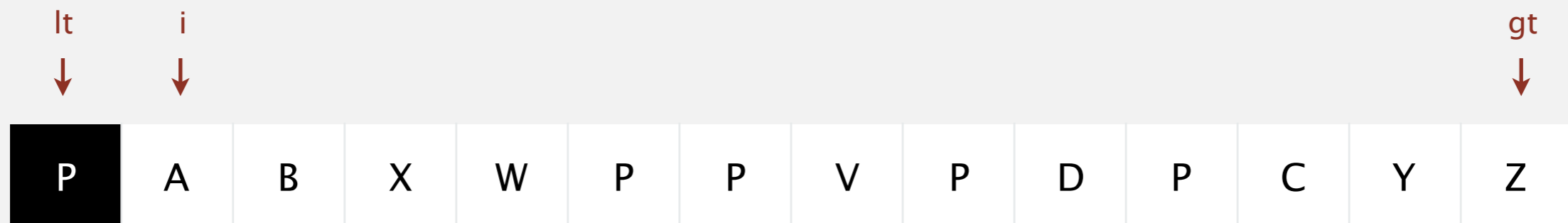
invariant



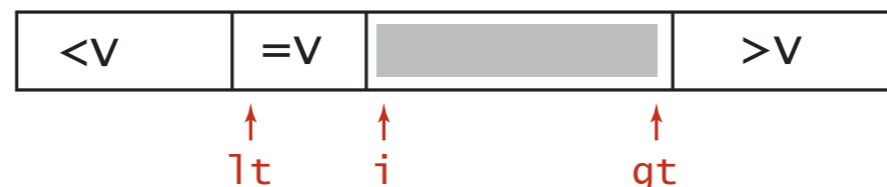
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- Let  $v$  be partitioning item  $a[l_0]$ .
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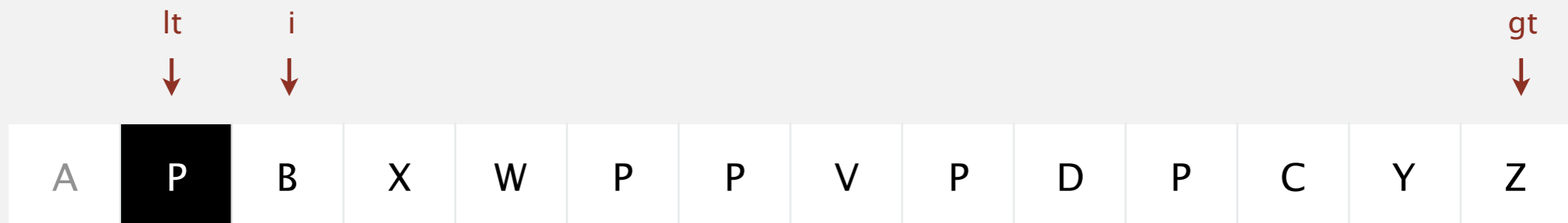
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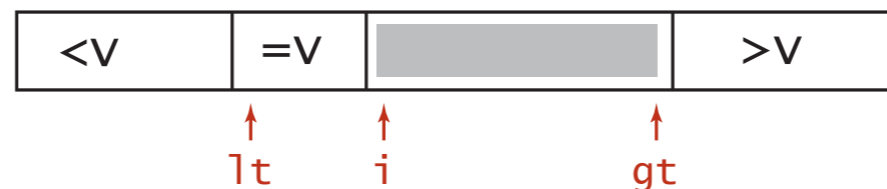
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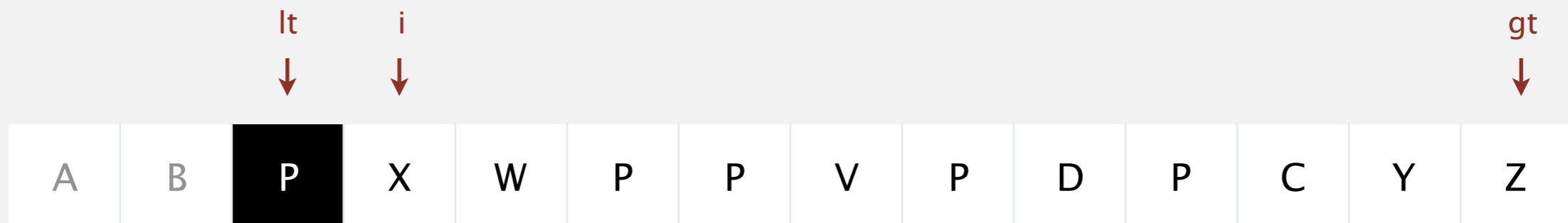
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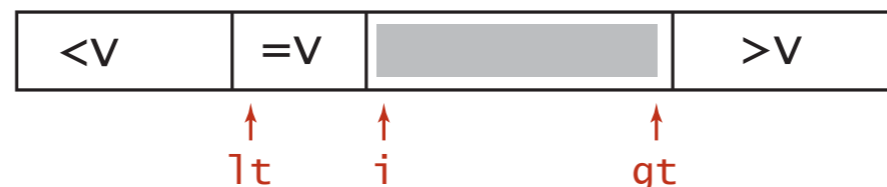
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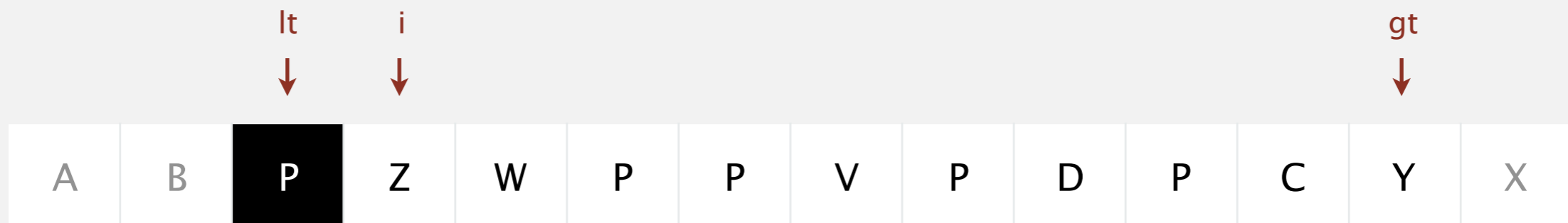
invariant



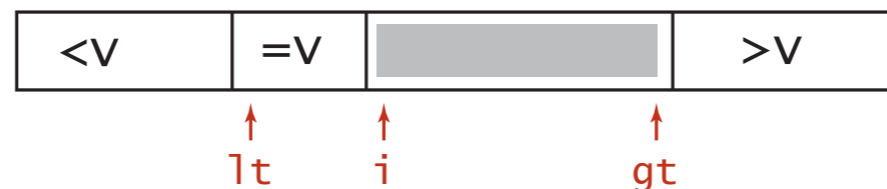
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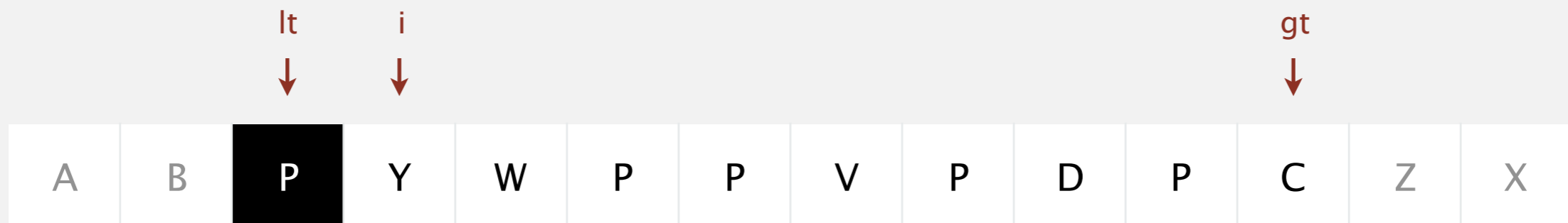
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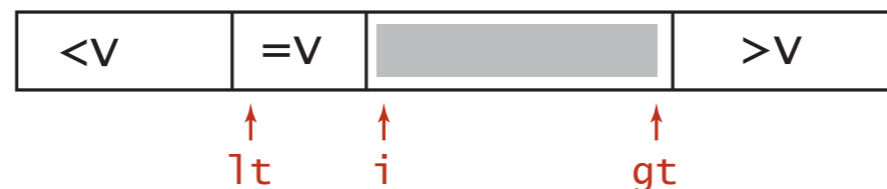
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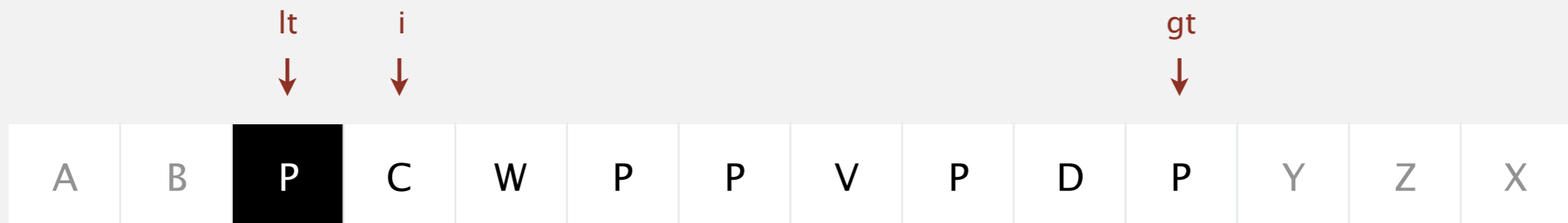
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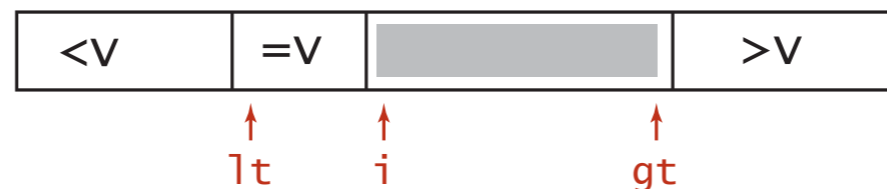
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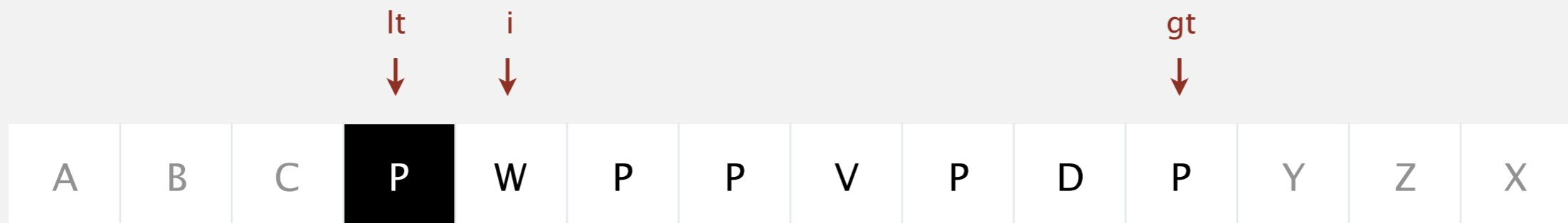
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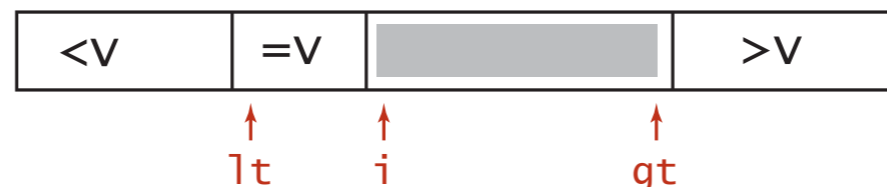
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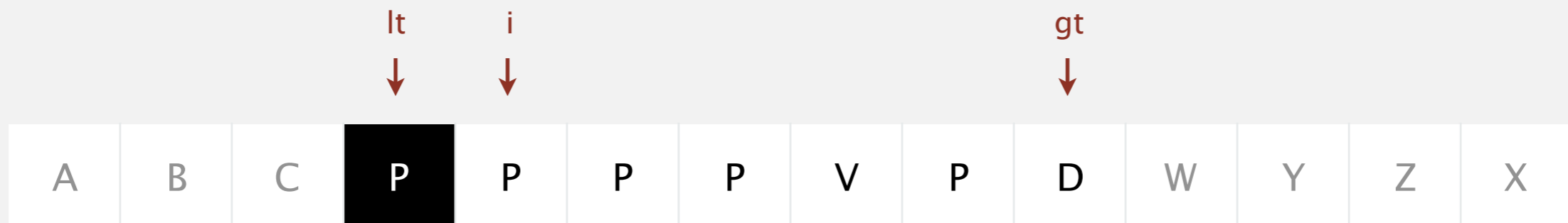




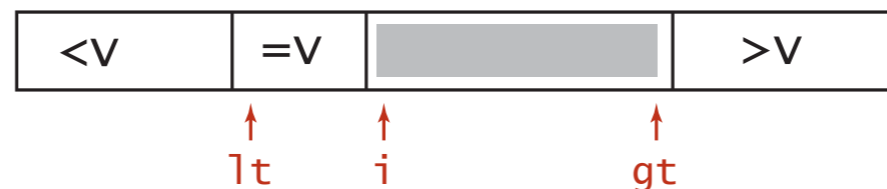
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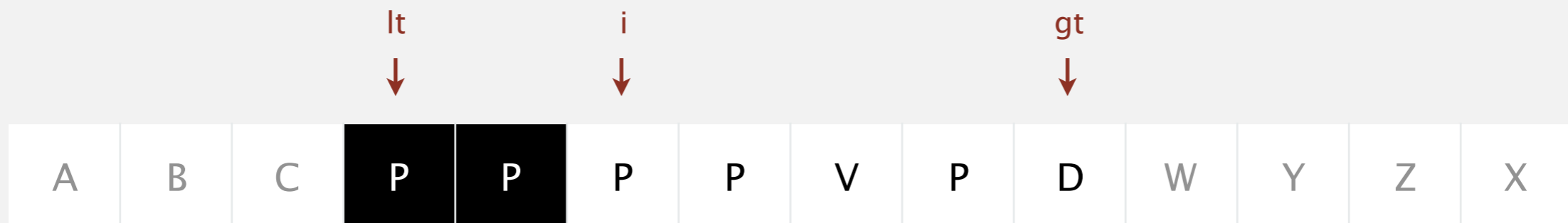
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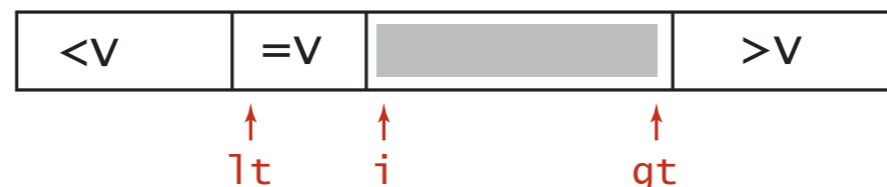
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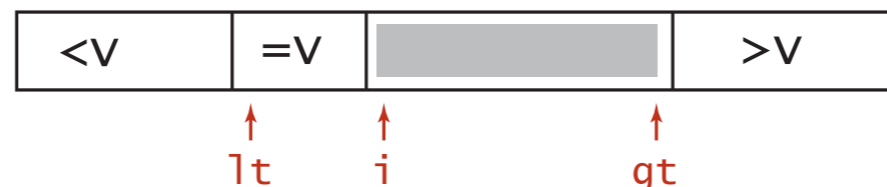
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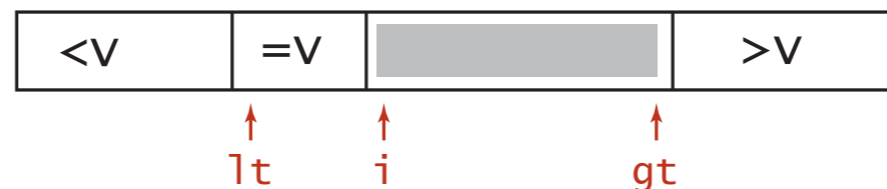
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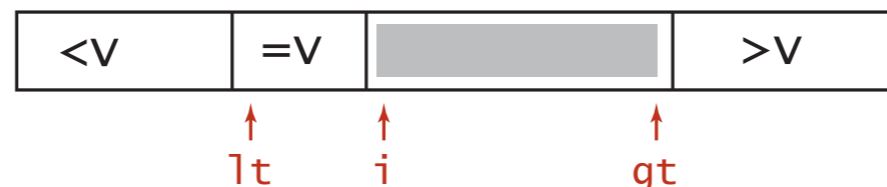
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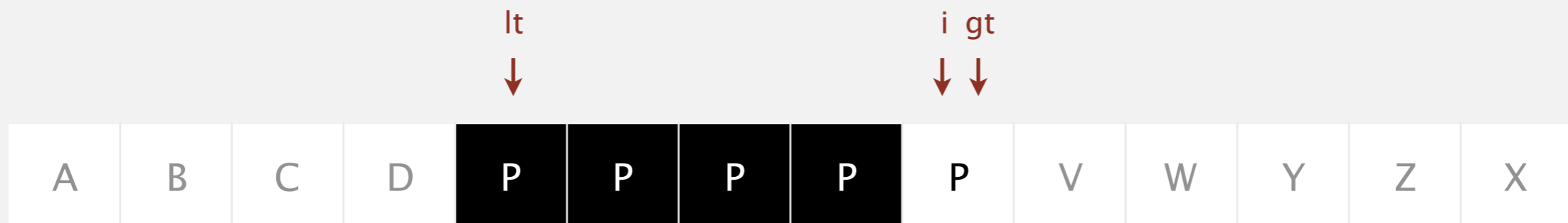
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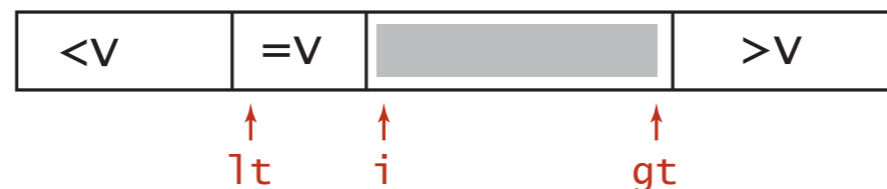
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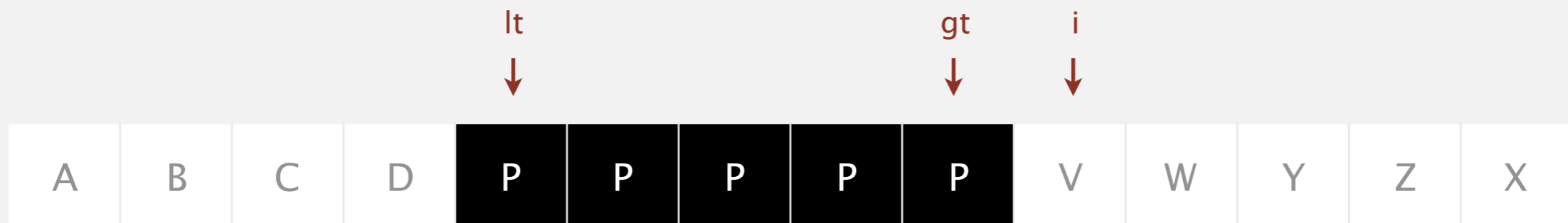
invariant



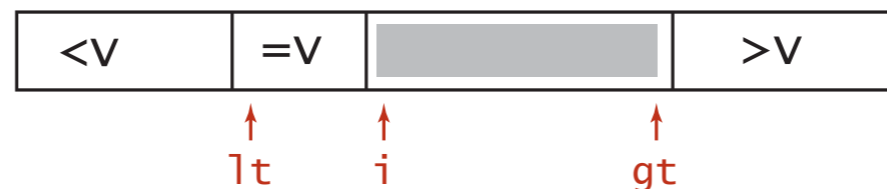
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[l_0]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[l_t]$  with  $a[i]$ ; increment both  $l_t$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



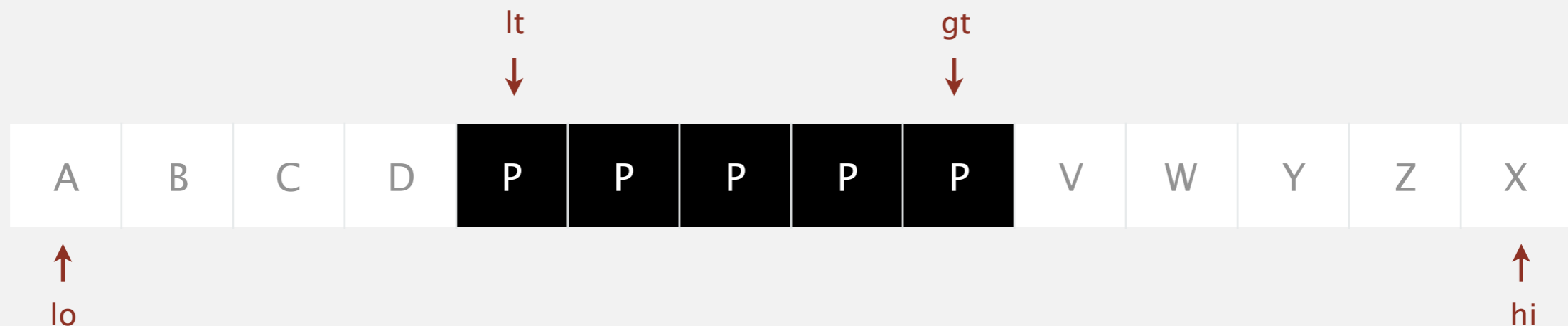
invariant



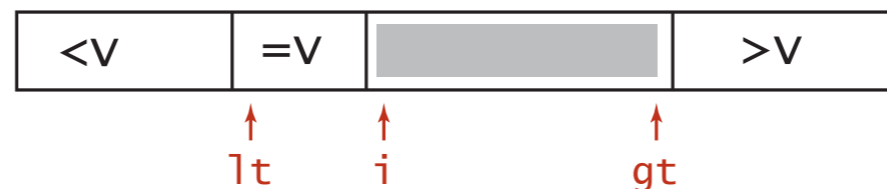
# Dijkstra 3-way partitioning demo

---

- Let  $v$  be partitioning item  $a[lo]$ .
- Scan  $i$  from left to right.
  - ( $a[i] < v$ ): exchange  $a[lt]$  with  $a[i]$ ; increment both  $lt$  and  $i$
  - ( $a[i] > v$ ): exchange  $a[gt]$  with  $a[i]$ ; decrement  $gt$
  - ( $a[i] == v$ ): increment  $i$



invariant







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## 2.3 PARTITIONING DEMOS

---

- *Sedgwick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

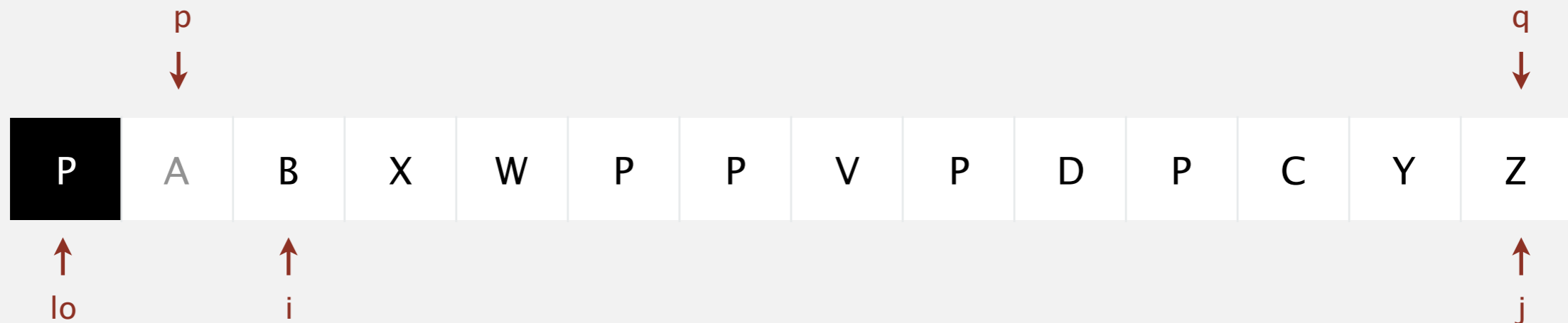


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

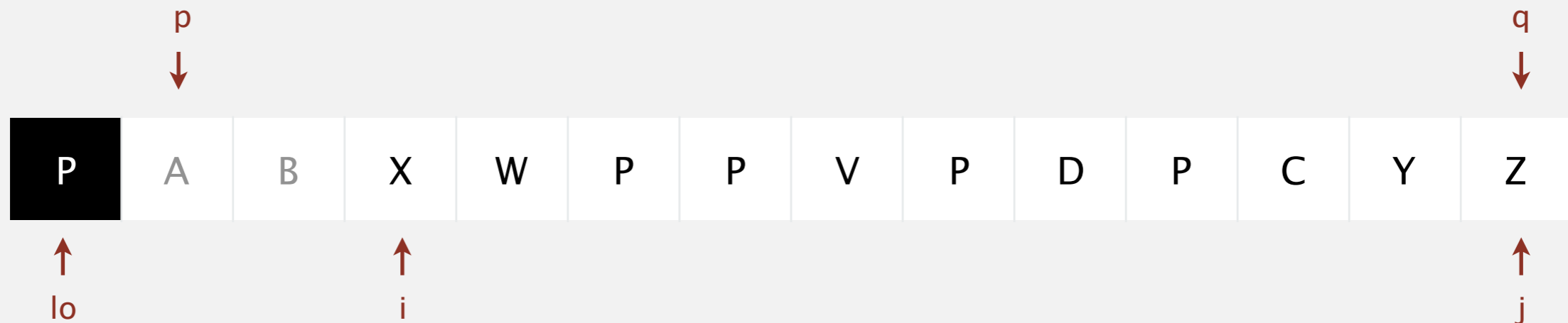


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

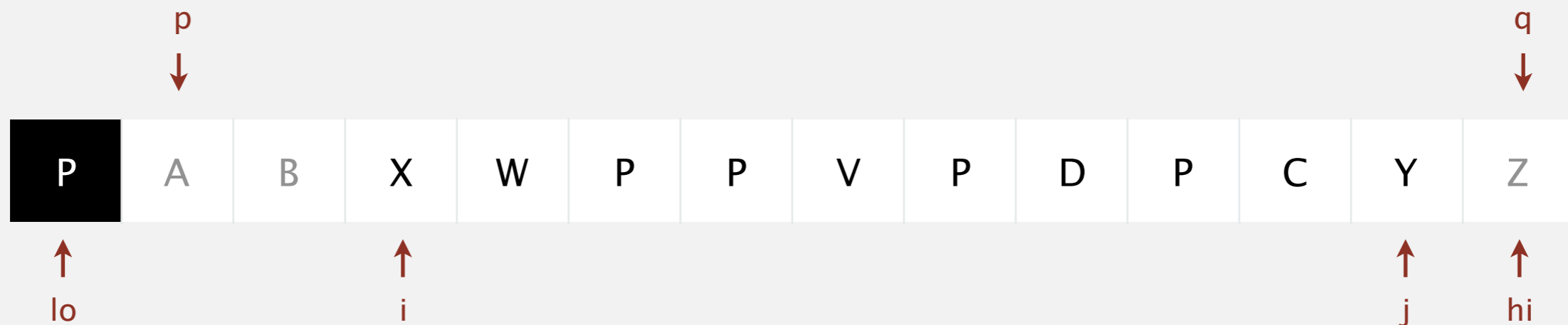


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

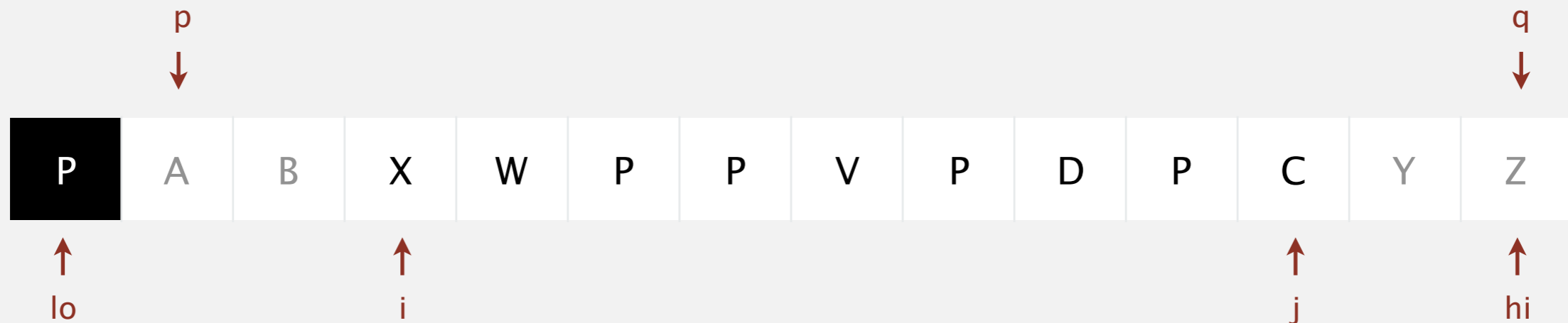


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



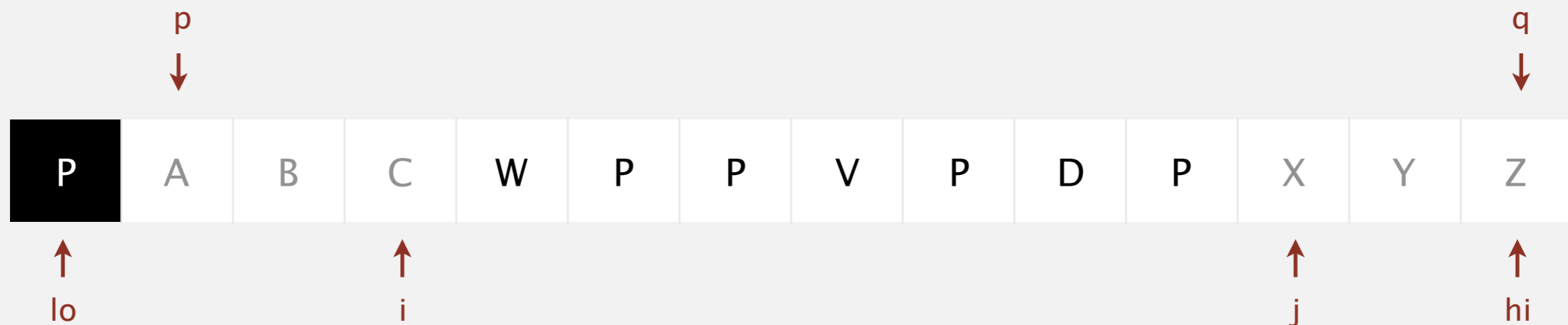
exchange  $a[i]$  with  $a[j]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



# Bentley-McIlroy 3-way partitioning demo

---

Phase I. Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



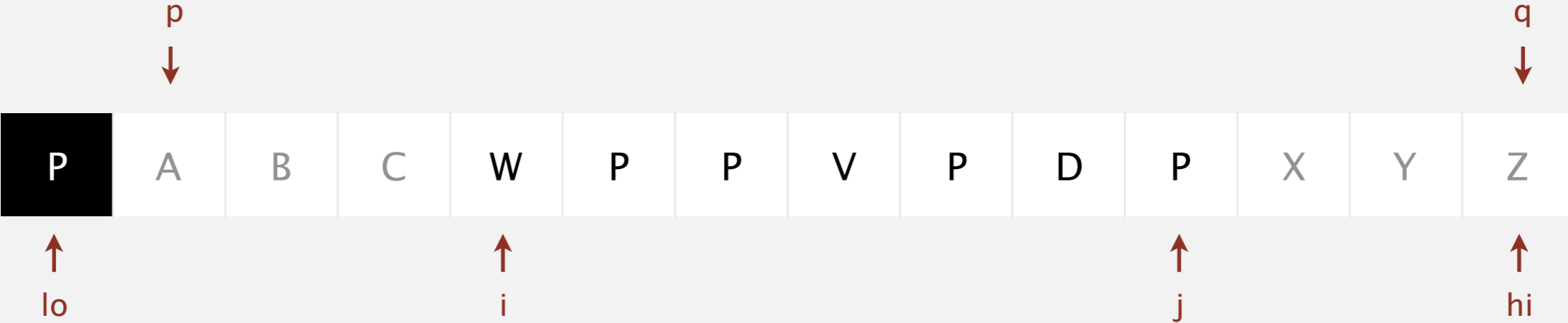


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until *i* and *j* pointers cross.

- Scan *i* from left to right so long as  $(a[i] < a[lo])$ .
- Scan *j* from right to left so long as  $(a[i] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment *p*.
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement *q*.



exchange  $a[i]$  with  $a[j]$

# Bentley-McIlroy 3-way partitioning demo

---

Phase I. Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



exchange  $a[i]$  with  $a[p]$  and increment  $p$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

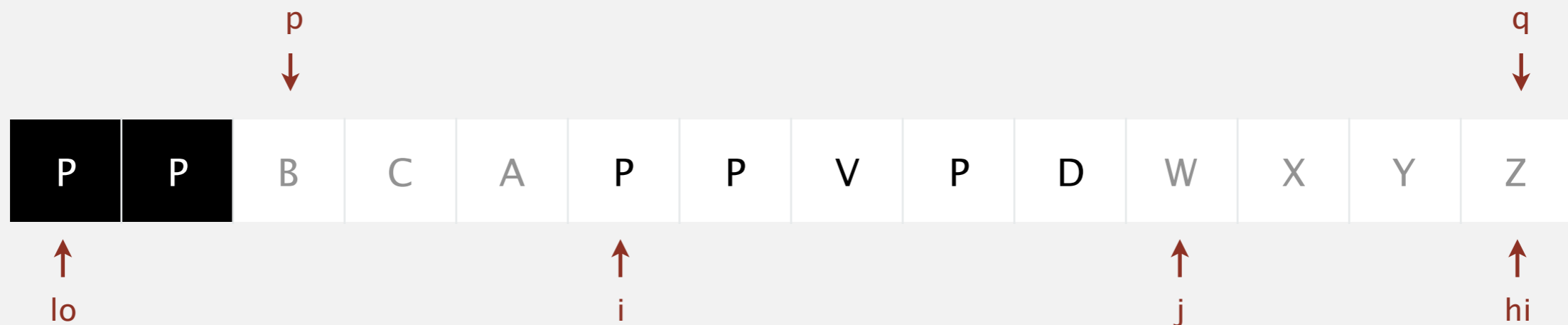


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

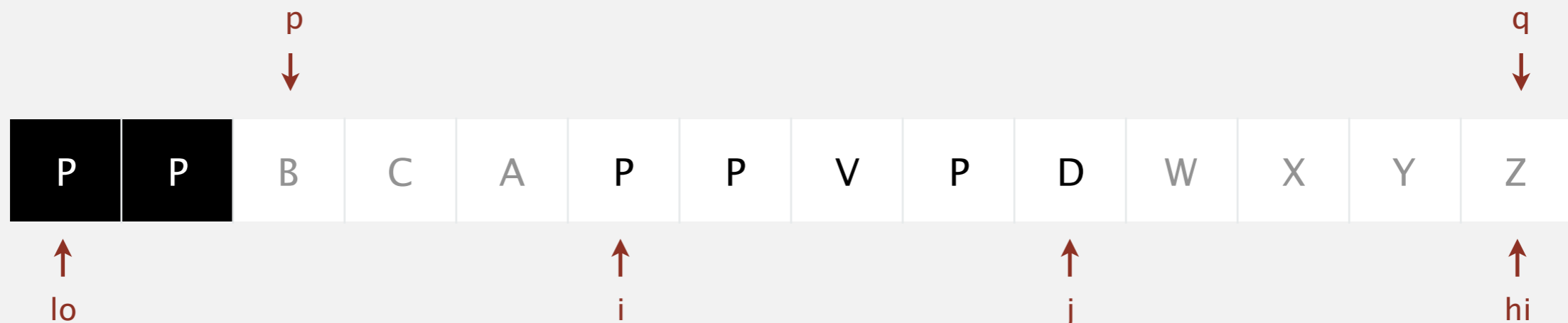


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



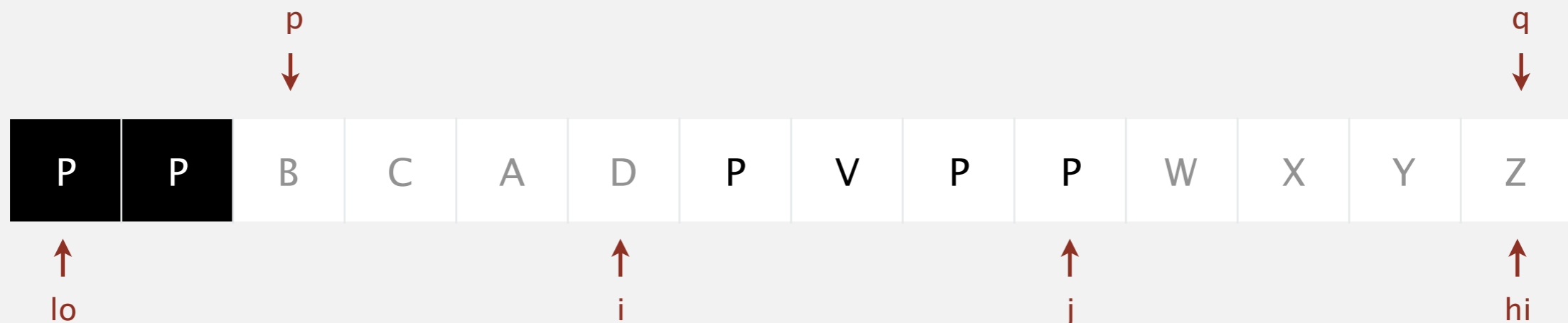
exchange  $a[i]$  with  $a[j]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



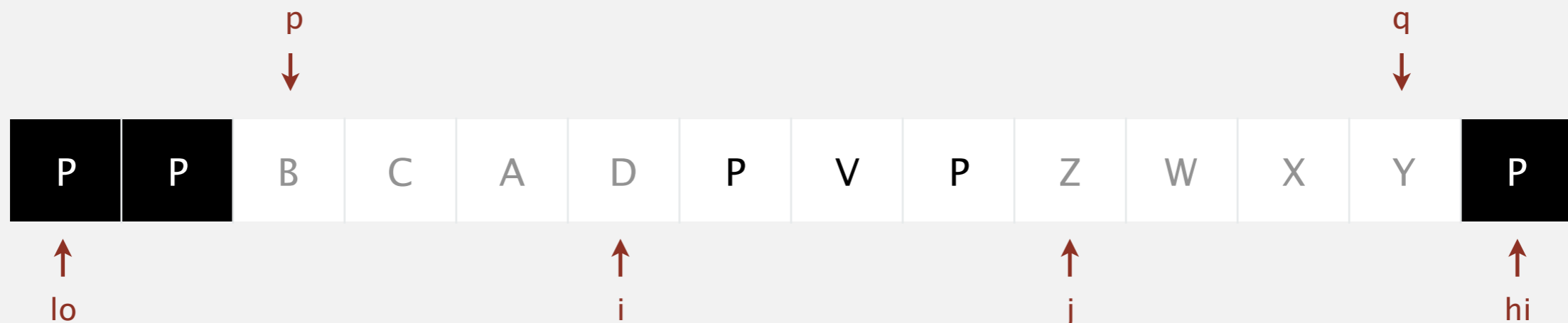
exchange  $a[j]$  with  $a[q]$  and decrement  $q$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .

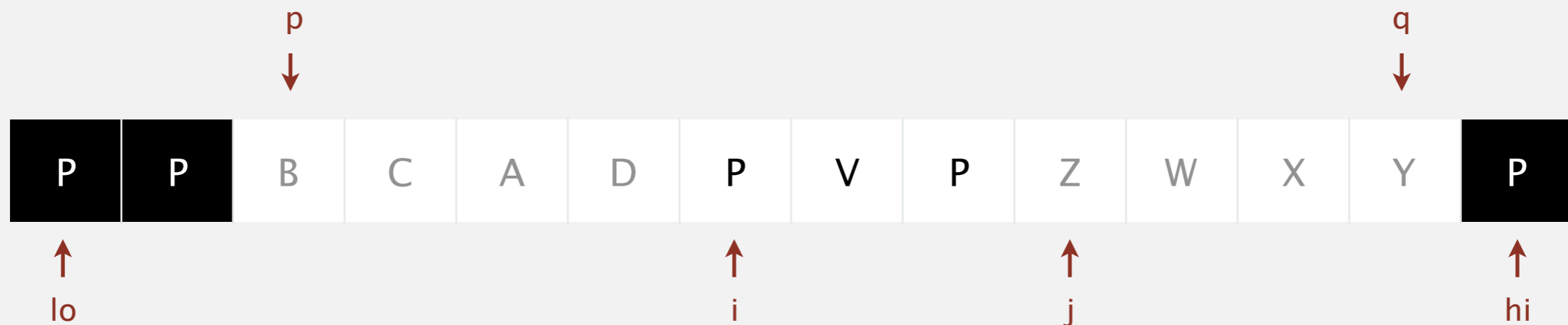


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



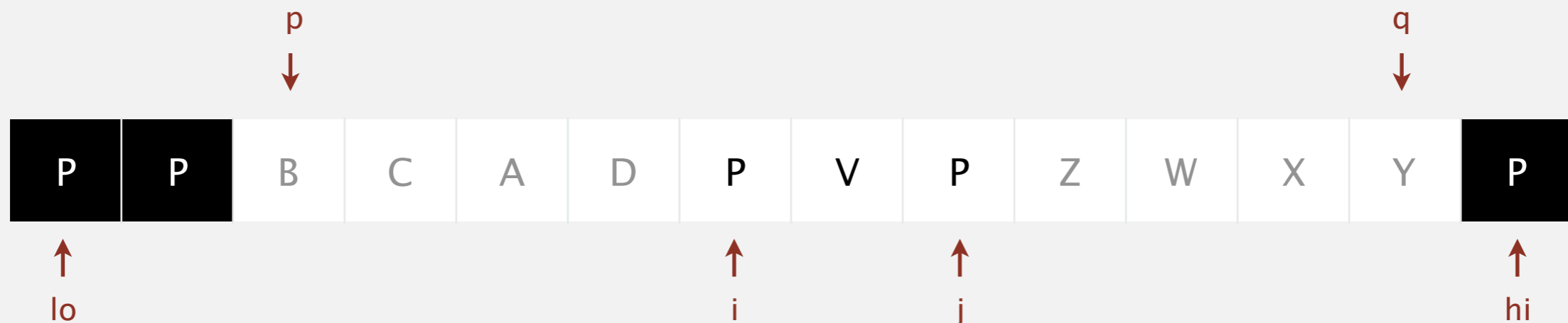


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



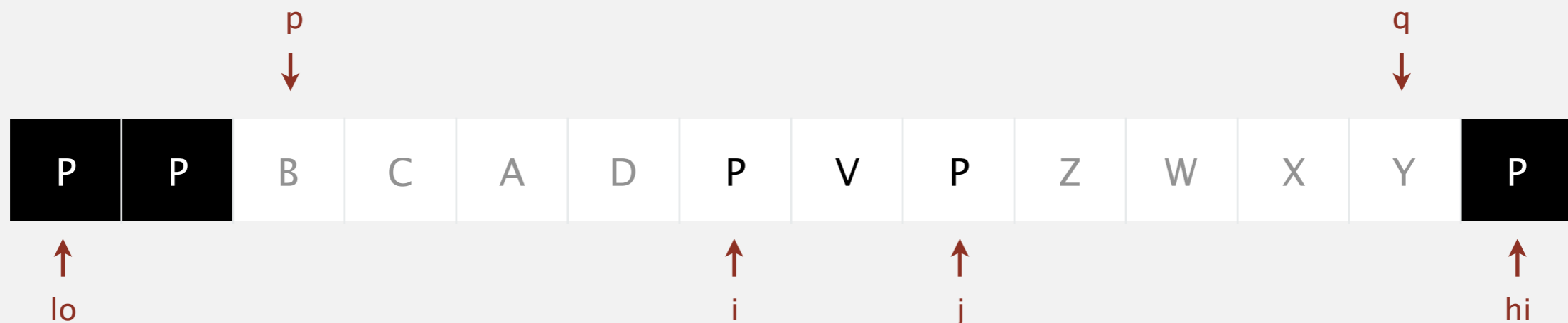
exchange  $a[i]$  with  $a[j]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
- If  $(a[j] == a[lo])$ , exchange  $a[j]$  with  $a[q]$  and decrement  $q$ .



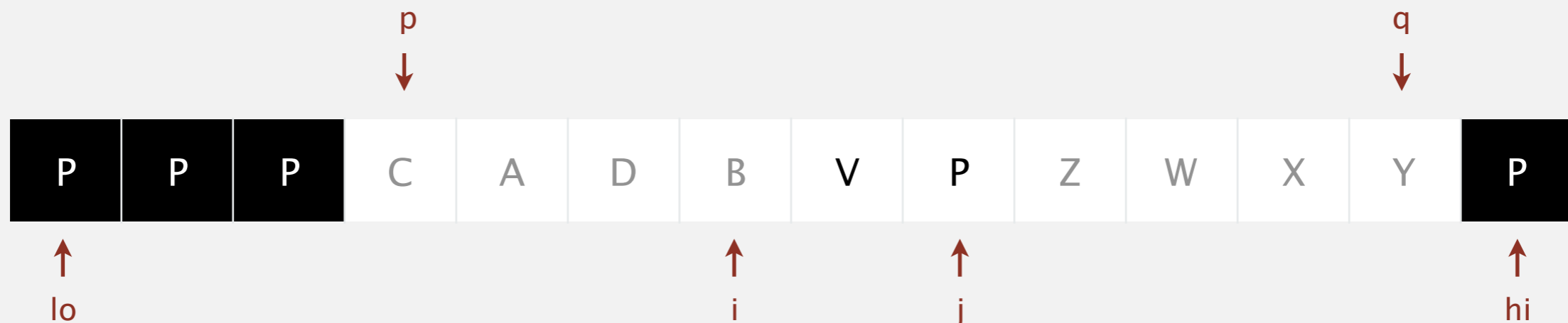
exchange  $a[i]$  with  $a[p]$  and increment  $p$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
- Scan  $j$  from right to left so long as  $(a[j] > a[lo])$ .
- Exchange  $a[i]$  with  $a[j]$ .
- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
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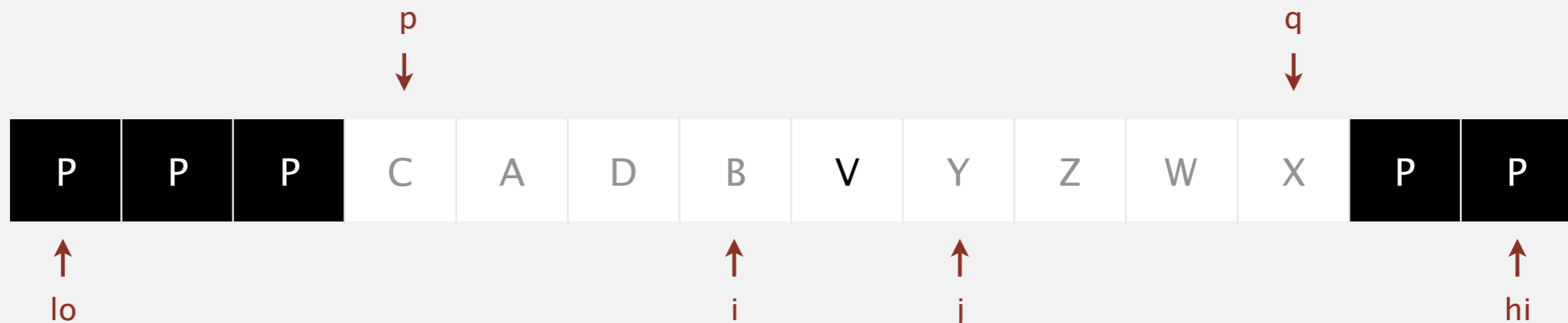
exchange  $a[j]$  with  $a[q]$  and decrement  $q$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
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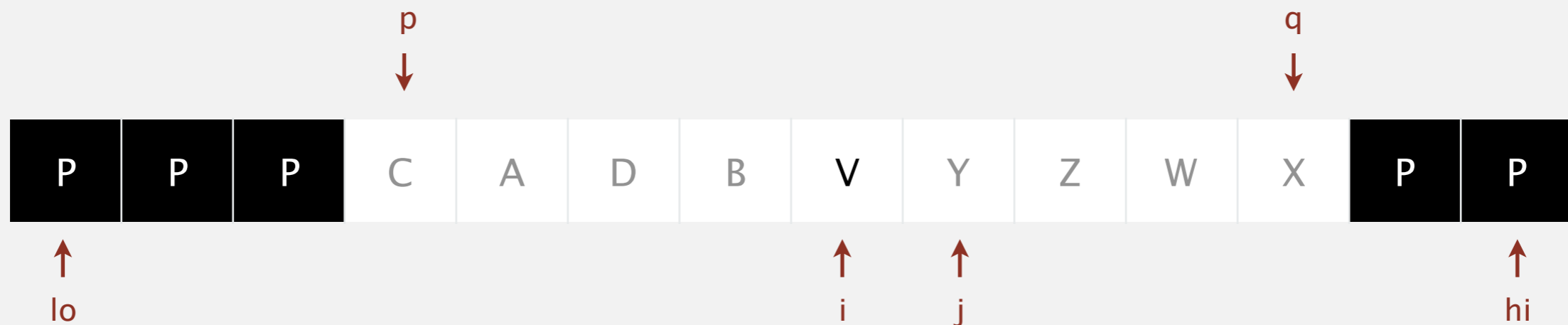


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
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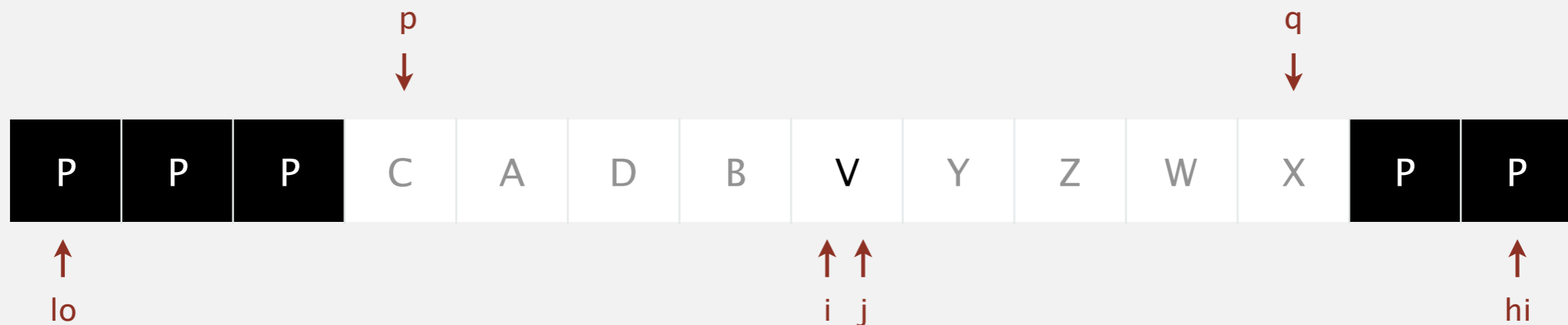


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
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- If  $(a[i] == a[lo])$ , exchange  $a[i]$  with  $a[p]$  and increment  $p$ .
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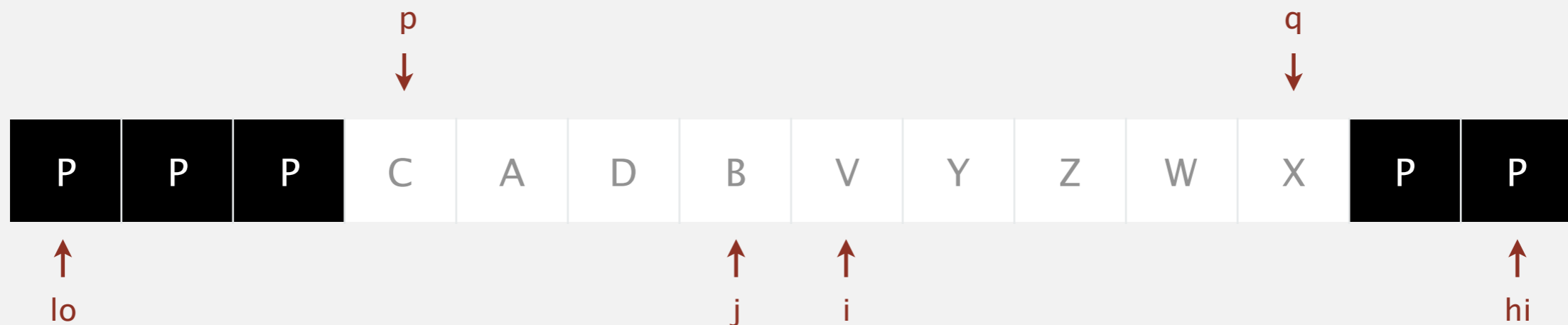


# Bentley-McIlroy 3-way partitioning demo

---

**Phase I.** Repeat until  $i$  and  $j$  pointers cross.

- Scan  $i$  from left to right so long as  $(a[i] < a[lo])$ .
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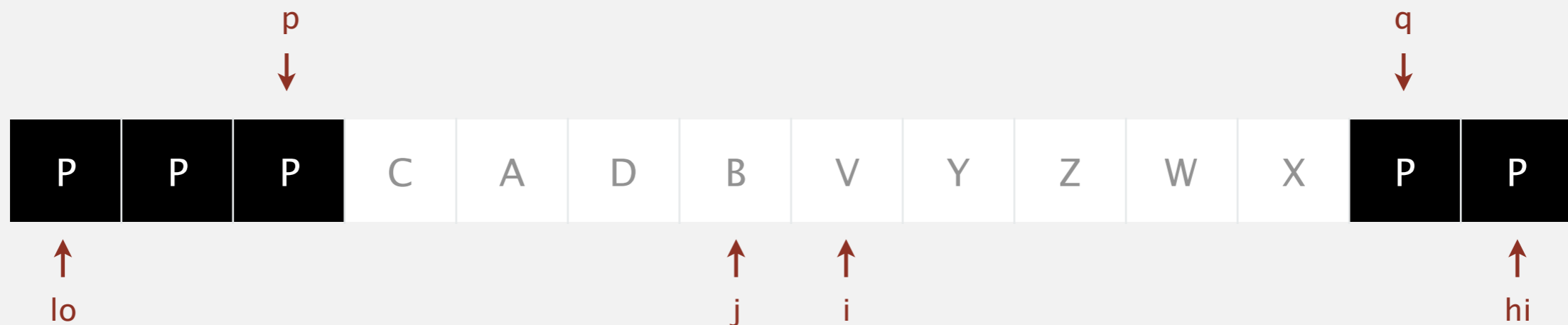
pointers cross

# Bentley-McIlroy 3-way partitioning demo

---

Phase II. Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



exchange  $a[j]$  with  $a[p]$



# Bentley-McIlroy 3-way partitioning demo

---

**Phase II.** Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



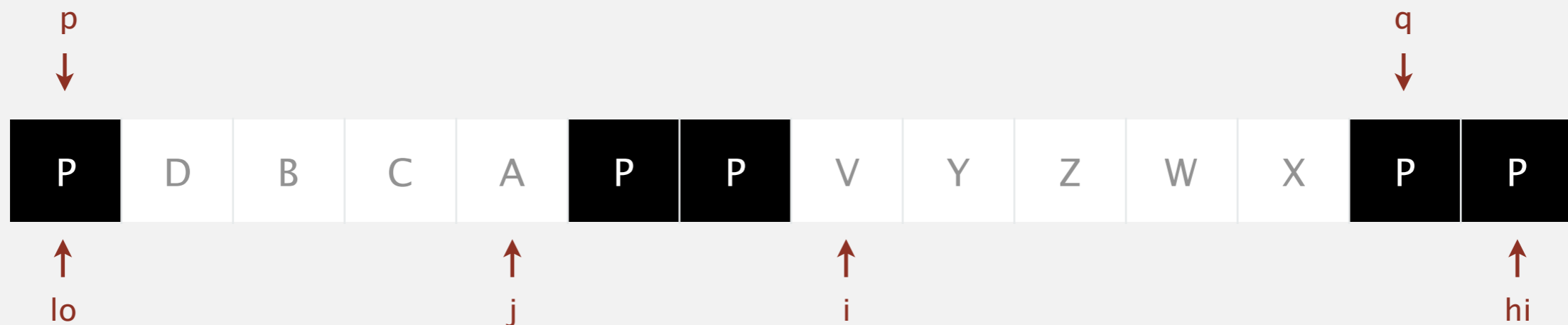
exchange  $a[j]$  with  $a[p]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase II.** Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



exchange  $a[j]$  with  $a[p]$

# Bentley-McIlroy 3-way partitioning demo

---

Phase II. Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



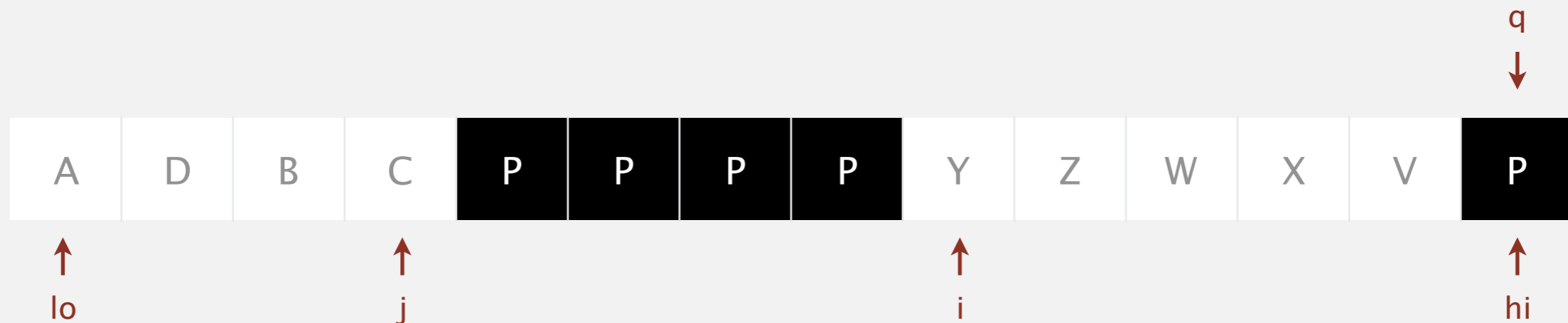
exchange  $a[i]$  with  $a[q]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase II.** Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



exchange  $a[i]$  with  $a[q]$

# Bentley-McIlroy 3-way partitioning demo

---

**Phase II.** Swap equal keys to the center.

- Scan  $j$  and  $p$  from right to left and exchange  $a[j]$  with  $a[p]$ .
- Scan  $i$  and  $q$  from left to right and exchange  $a[i]$  with  $a[q]$ .



3-way partitioned



<http://algs4.cs.princeton.edu>

## 2.3 PARTITIONING DEMOS

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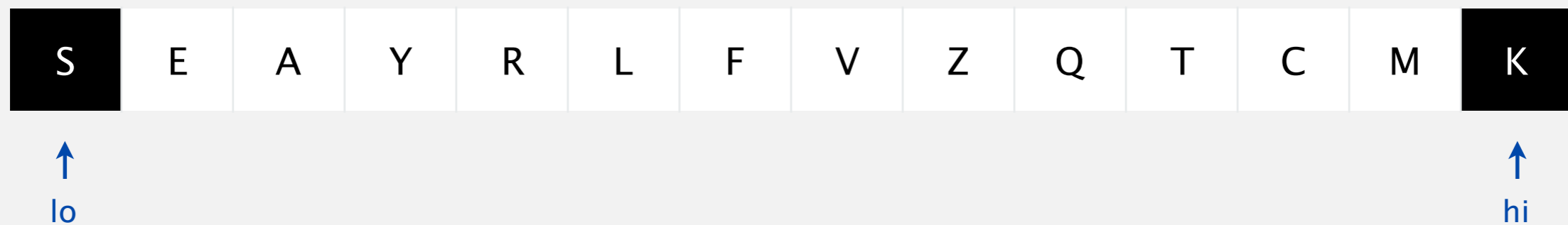
- *Sedgwick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*

# Dual-pivot partitioning demo

---

## Initialization.

- Choose  $a[lo]$  and  $a[hi]$  as partitioning items.
- Exchange if necessary to ensure  $a[lo] \leq a[hi]$ .



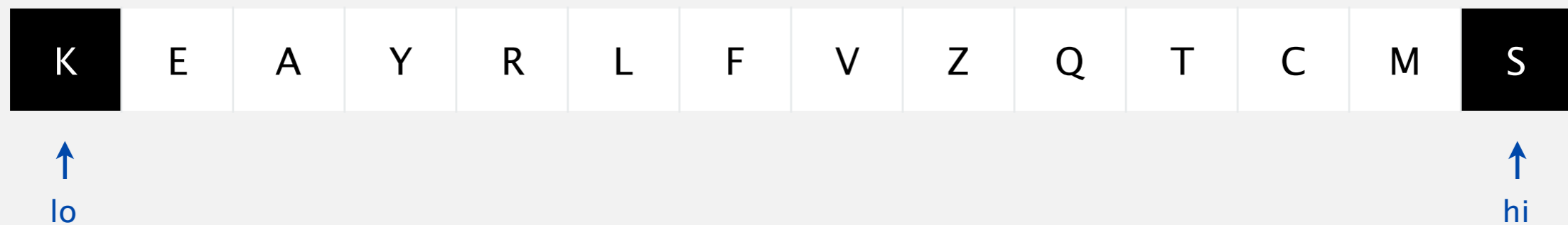
exchange  $a[lo]$  and  $a[hi]$

# Dual-pivot partitioning demo

---

## Initialization.

- Choose  $a[lo]$  and  $a[hi]$  as partitioning items.
- Exchange if necessary to ensure  $a[lo] \leq a[hi]$ .



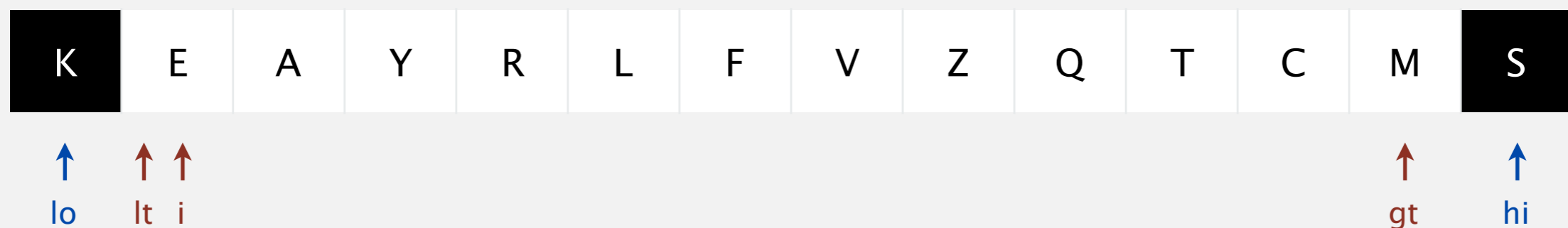
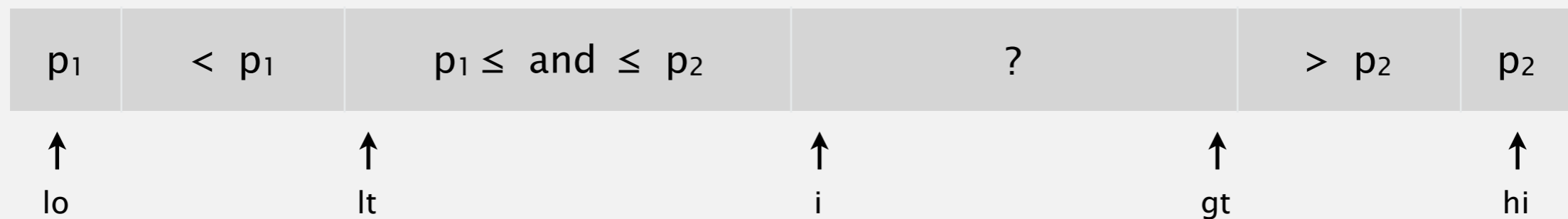


# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



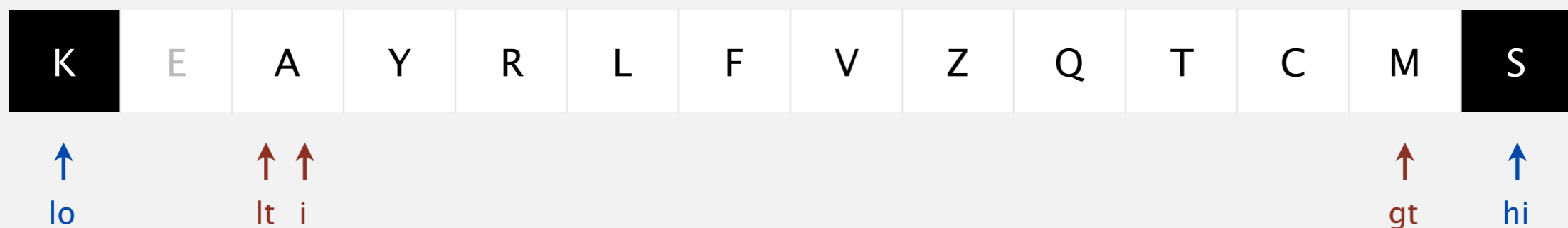
**exchange  $a[i]$  and  $a[lt]$ ; increment  $lt$  and  $i$**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



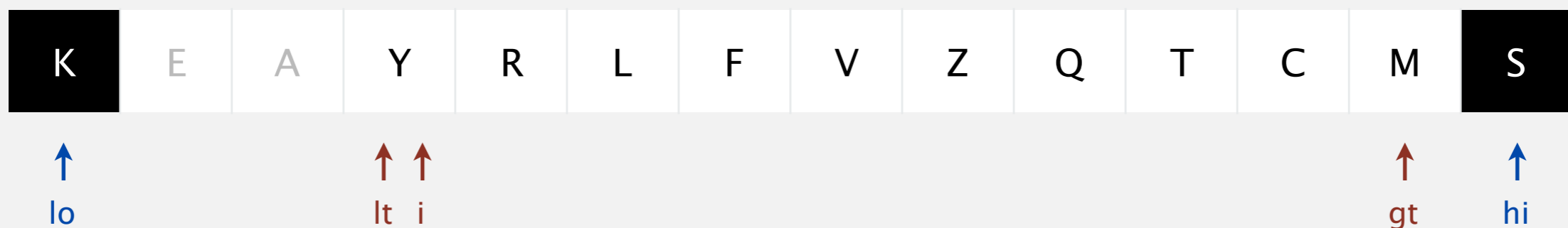
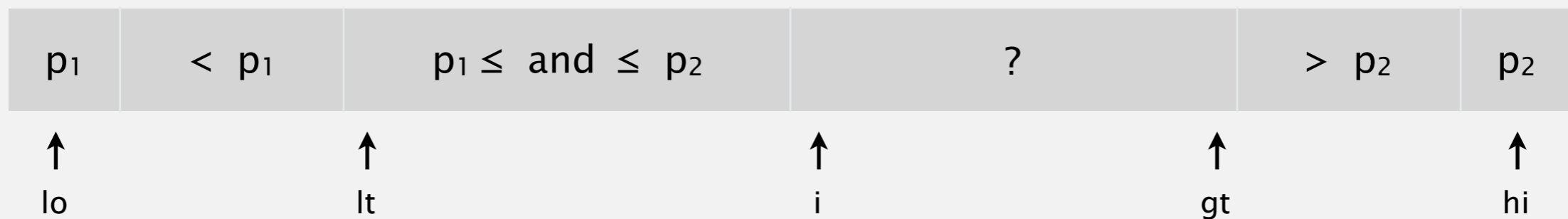
**exchange  $a[i]$  and  $a[lt]$ ; increment  $lt$  and  $i$**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



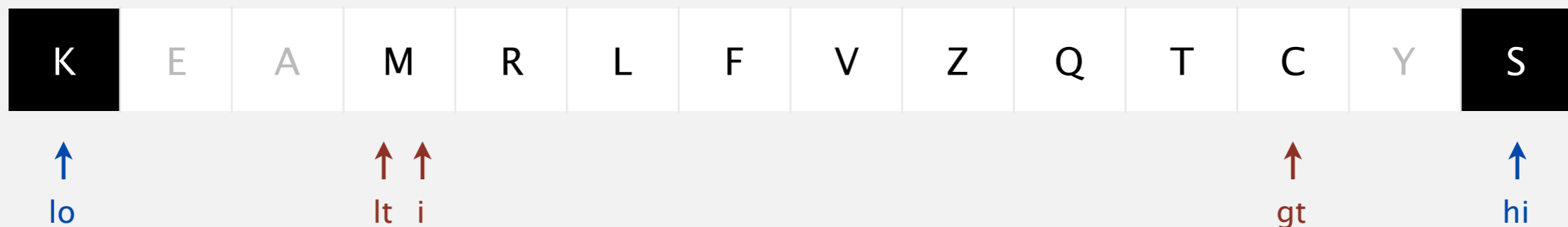
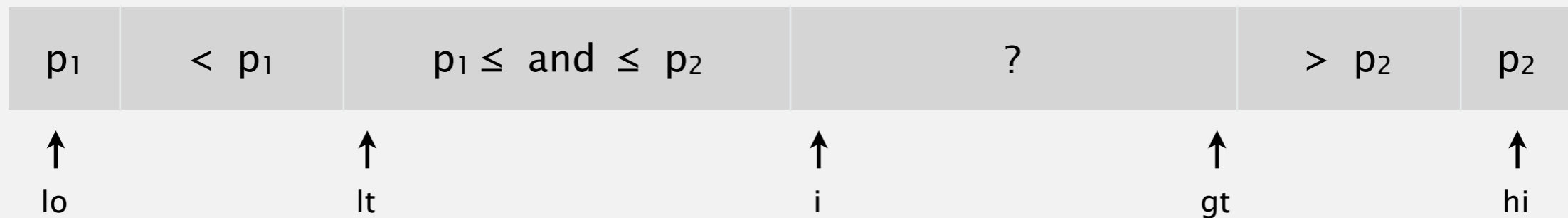
**exchange  $a[i]$  and  $a[gt]$ ; decrement  $gt$**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
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- Else, increment  $i$ .



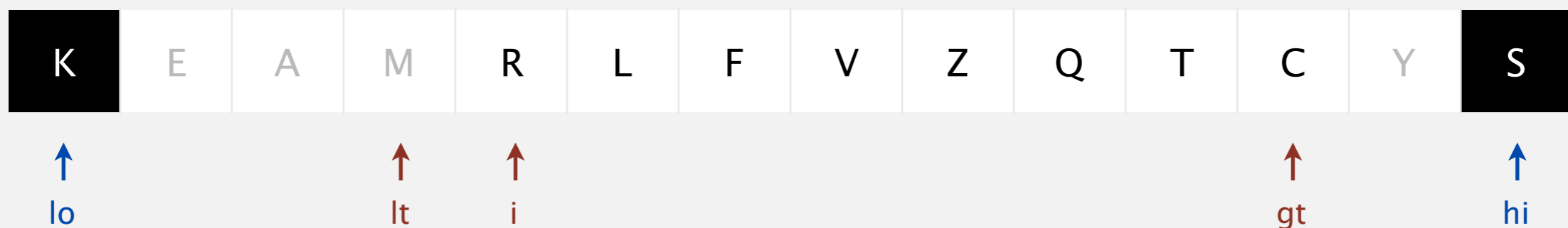
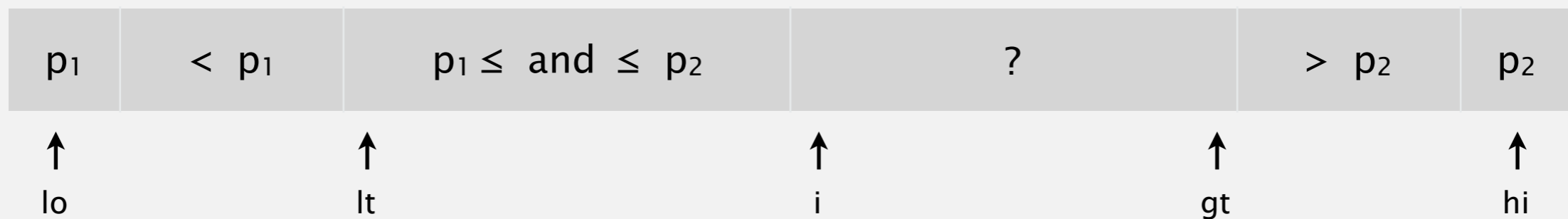
**increment i**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
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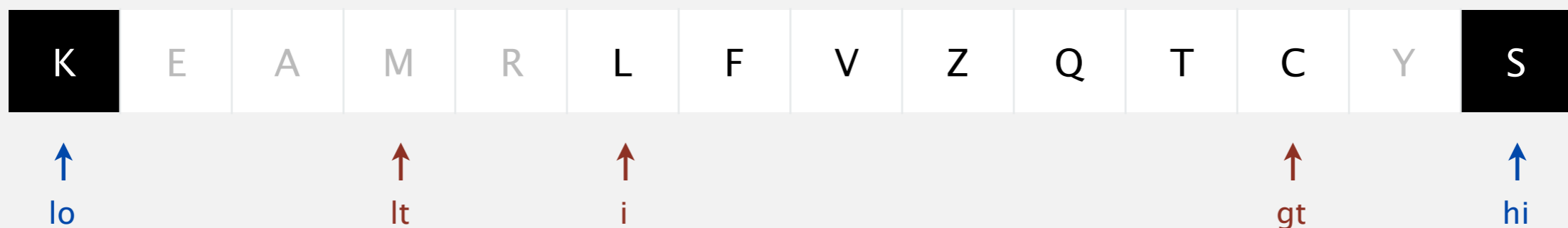
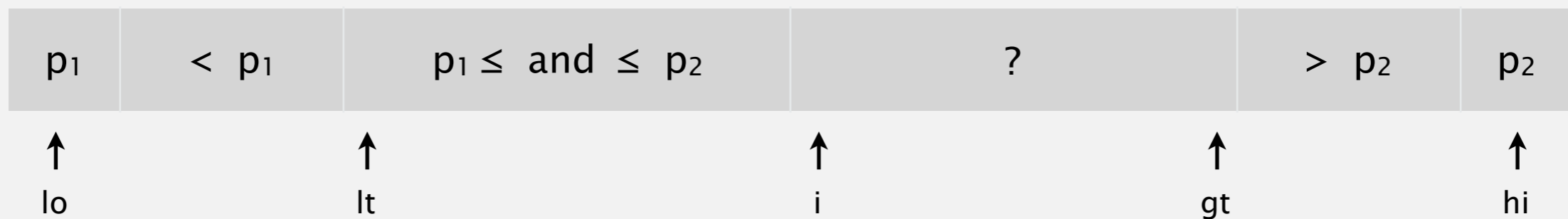
**increment i**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
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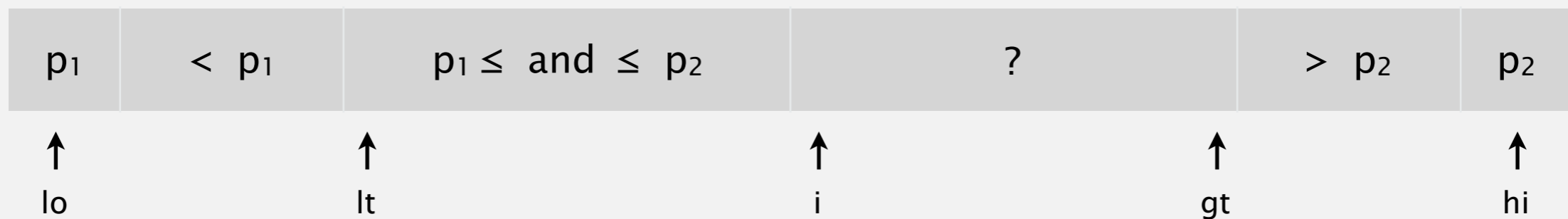
**increment i**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[l_t]$  and increment  $l_t$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



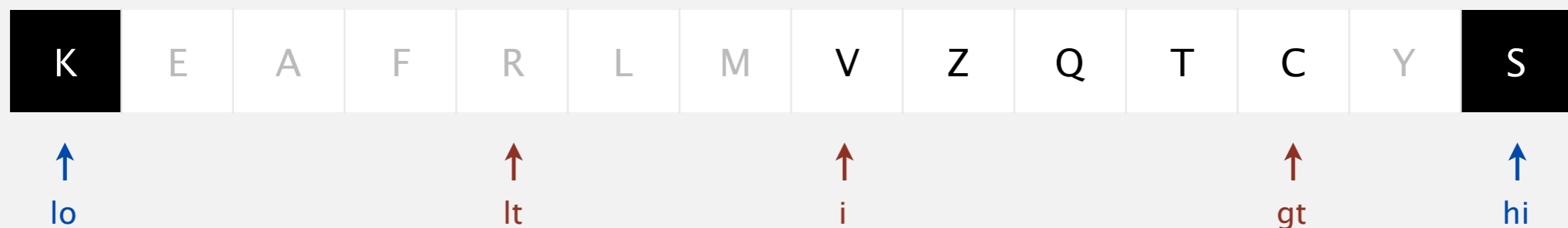
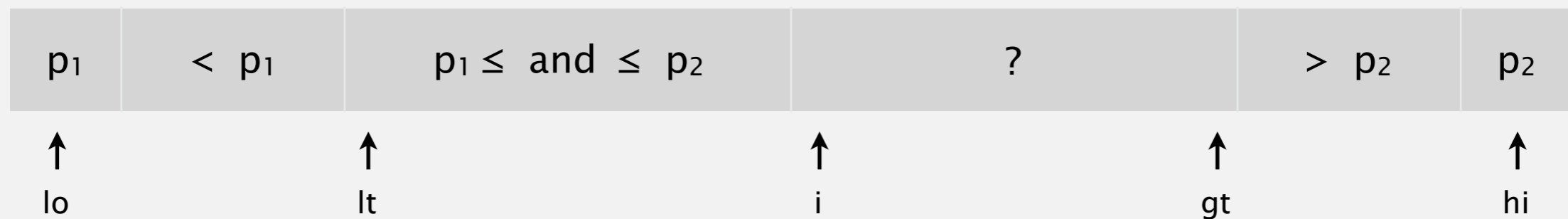
**exchange  $a[i]$  and  $a[l_t]$ ; increment  $l_t$  and  $i$**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



**exchange  $a[i]$  and  $a[gt]$ ; decrement  $gt$**

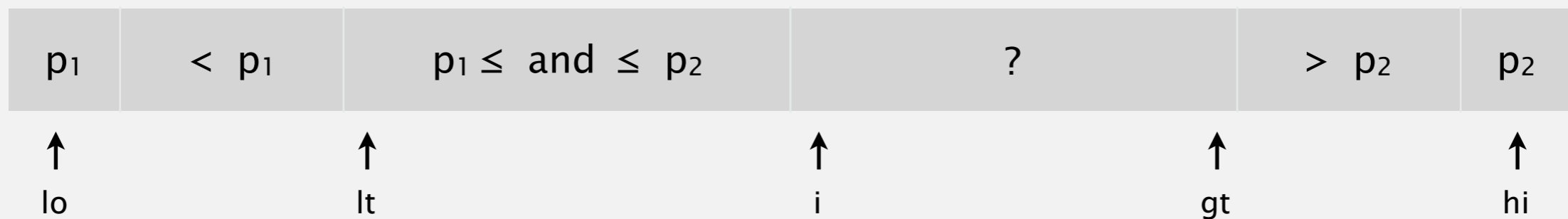


# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
- Else, increment  $i$ .



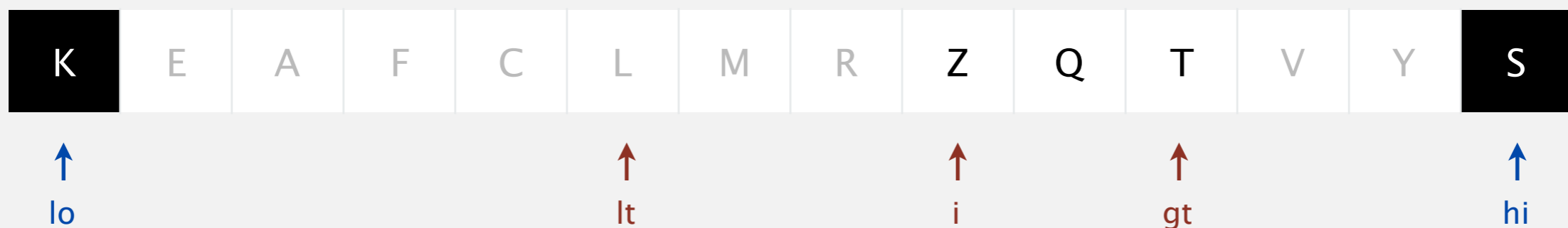
**exchange  $a[i]$  and  $a[lt]$ ; increment  $lt$  and  $i$**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
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- Else, increment  $i$ .



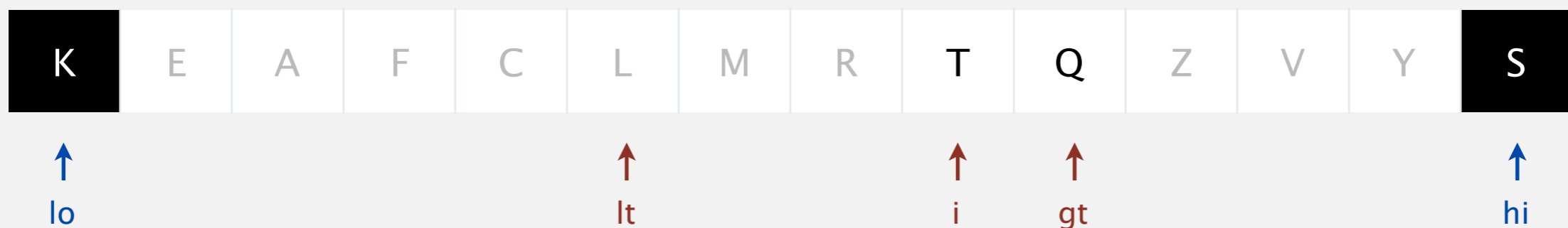
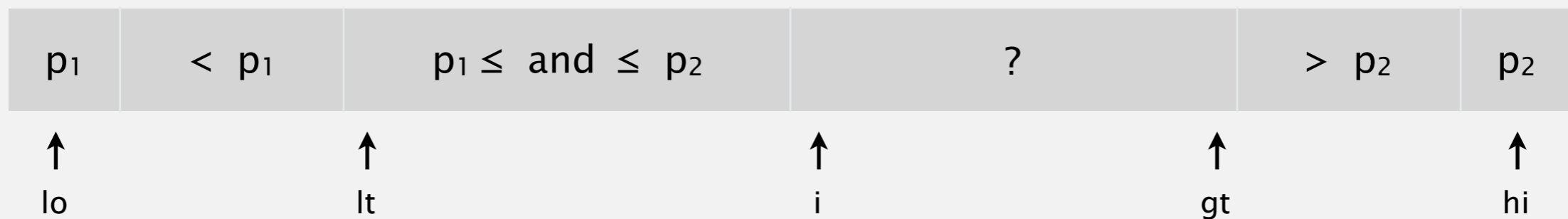
**exchange  $a[i]$  and  $a[gt]$ ; decrement  $gt$**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
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- Else, increment  $i$ .



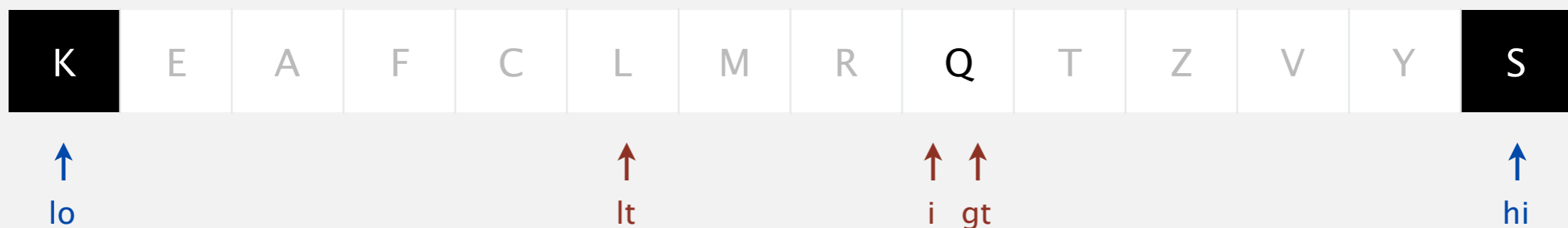
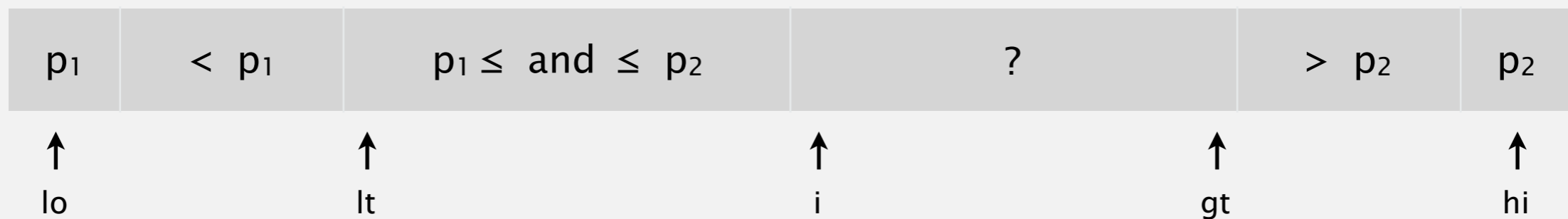
**exchange  $a[i]$  and  $a[gt]$ ; decrement  $gt$**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
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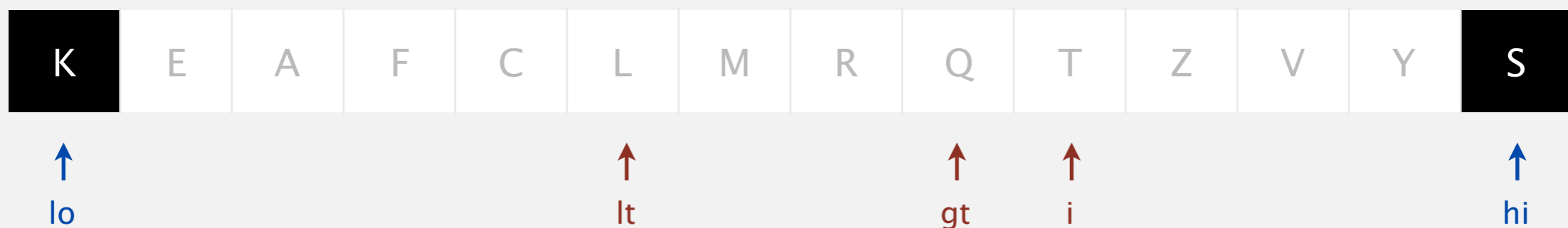
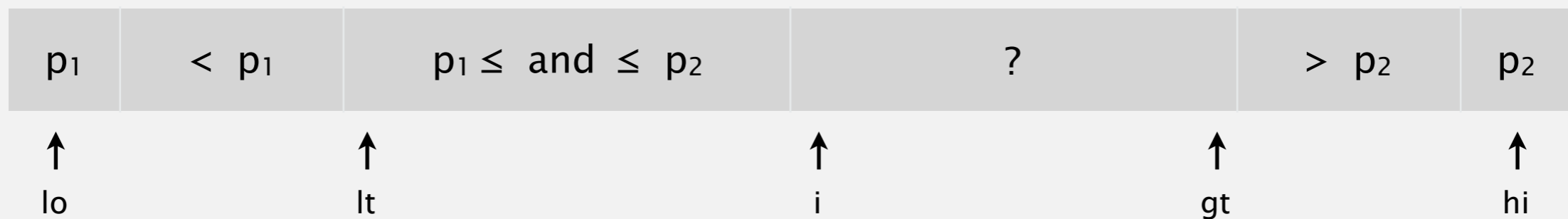
**increment i**

# Dual-pivot partitioning demo

---

**Main loop.** Repeat until  $i$  and  $gt$  pointers cross.

- If  $(a[i] < a[lo])$ , exchange  $a[i]$  with  $a[lt]$  and increment  $lt$  and  $i$ .
- Else if  $(a[i] > a[hi])$ , exchange  $a[i]$  with  $a[gt]$  and decrement  $gt$ .
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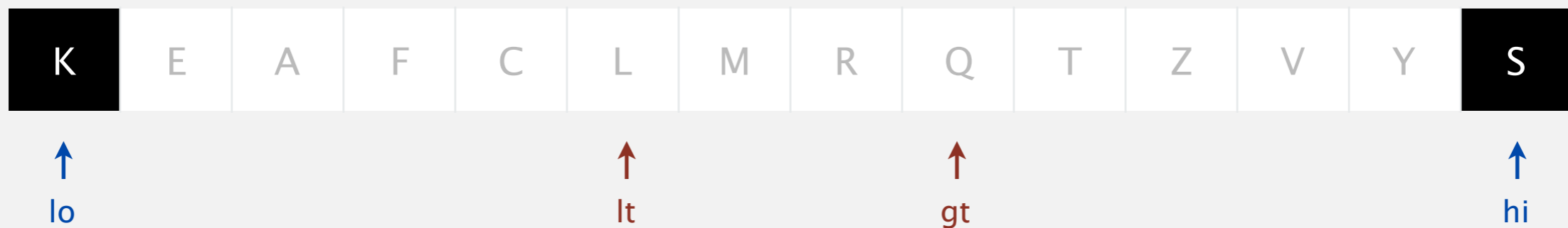
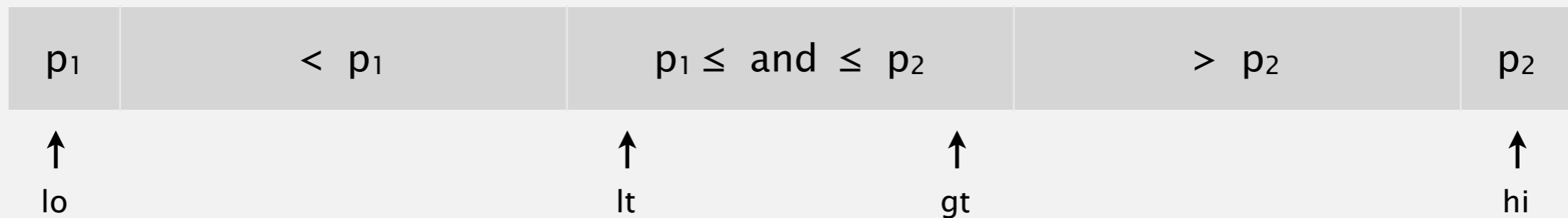
**stop when pointers cross**

# Dual-pivot partitioning demo

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## Finalize.

- Exchange  $a[lo]$  with  $a[--lt]$ .
- Exchange  $a[hi]$  with  $a[++gt]$ .

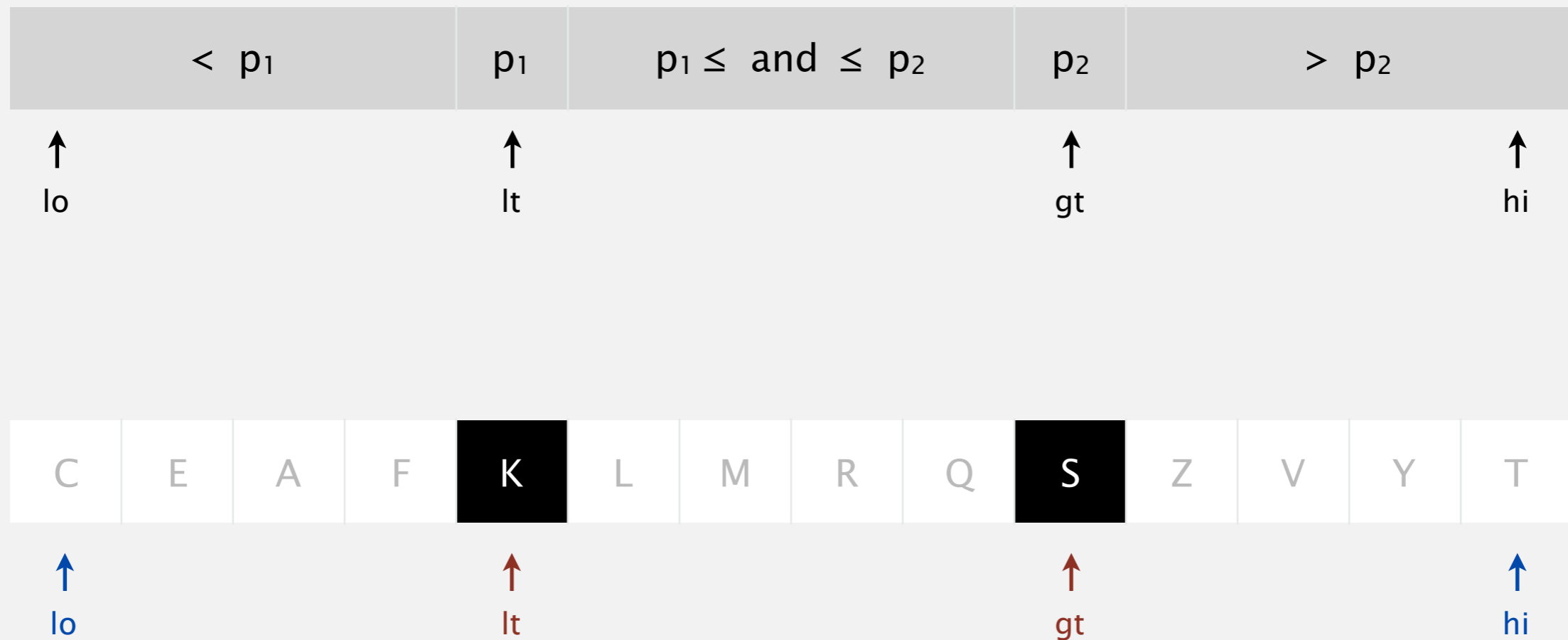


# Dual-pivot partitioning demo

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## Finalize.

- Exchange  $a[lo]$  with  $a[--lt]$ .
- Exchange  $a[hi]$  with  $a[++gt]$ .



**3-way partitioned**