Constituency Parsing Data structures and algorithms for Computational Linguistics III

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Context free grammars

recap

- Context free grammars are sufficient for expressing most phenomena in natural language syntax
- Most of the parsing theory (and quite some of the practice) is build on parsing CF languages
- The context-free rules have the form

$$A \rightarrow \alpha$$

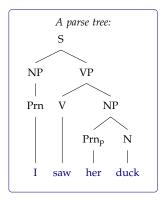
where A is a single non-terminal symbol and α is a (possibly empty) sequence of terminal or non-terminal symbols

An example context-free grammar

```
Derivation of sentence 'she saw a duck'
S
      \rightarrow NP VP
      \rightarrow Aux NP VP
                                      \Rightarrow NP VP
NP \rightarrow Det N
                                NP \Rightarrow Prn
NP \rightarrow Prn
                                Prn \Rightarrow she
NP \rightarrow NP PP
                                                                 NP
                                                                                  VP
                                VP \Rightarrow V NP
VP \rightarrow V NP
                                V \Rightarrow saw
VP \rightarrow V
                                NP \Rightarrow Det N
                                                                 Prn
                                                                                        NP
VP \rightarrow VP PP
                                Det \Rightarrow a
PP \rightarrow Prp NP
                                N \Rightarrow duck
N \rightarrow duck
                                                                                   Det
N \rightarrow park
V \rightarrow duck
V \rightarrow ducks
                                                                                            duck
                                                                 she
                                                                          saw
                                                                                     a
    \rightarrow saw
Prn \rightarrow she \mid her
```

 $Prp \rightarrow in \mid with$

Representations of a context-free parse tree



A history of derivations:

- $S \Rightarrow NP VP$
- NP \Rightarrow Prn
- Prn \Rightarrow I
- VP ⇒V NP
- $V \Rightarrow saw$
 - $NP \Rightarrow Prn_p N$
- $Prn_p \Rightarrow her$
- $N \Rightarrow duck$

A sequence with (labeled) brackets

Parsing as search

- Parsing can be seen as search constrained by the grammar and the input
- Top down: start from S, find the derivations that lead to the sentence
- Bottom up: start from the sentence, find series of derivations (in reverse) that leads to S
- Search can be depth first or breadth first for both cases

 $S \rightarrow NP VP$ she saw duck

 \rightarrow Aux NP VP

 $NP \rightarrow Det N$

 $NP \rightarrow Prn$ $NP \rightarrow NP PP$

 $VP \rightarrow V NP$

 $VP \rightarrow V$

 $VP \rightarrow VP PP$

 $PP \rightarrow Prp NP$

 $N \rightarrow duck$

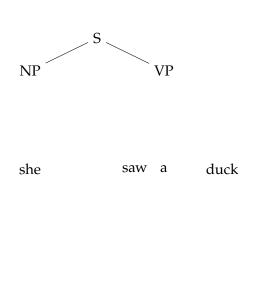
 $N \rightarrow park$

 $V \rightarrow duck$ $V \rightarrow ducks$

 \rightarrow saw

 $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$

Det \rightarrow a | the WS 18-19 5 / 29



 $\begin{array}{ll} S & \rightarrow \text{NP VP} \\ S & \rightarrow \text{Aux NP VP} \\ \hline \text{NP} & \rightarrow \text{Det N} \\ \hline \text{NP} & \rightarrow \text{Prn} \\ \hline \text{NP} & \rightarrow \text{NP PP} \\ \hline \text{VP} & \rightarrow \text{V NP} \\ \hline \text{VP} & \rightarrow \text{V} \end{array}$

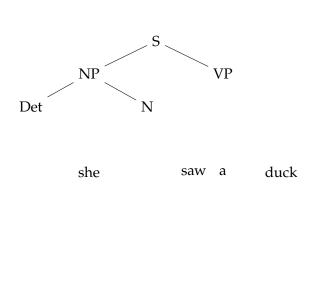
 $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$

 $egin{array}{ll} N &
ightarrow duck \ N &
ightarrow park \ V &
ightarrow duck \end{array}$

 $V \rightarrow duck$ $V \rightarrow ducks$

 $V \rightarrow saw$ Prn $\rightarrow she \mid her$

 $\begin{array}{c} \text{Prp} \rightarrow \text{in} \mid \text{with} \\ \text{Det} \rightarrow \text{a} \mid \text{the} \end{array}$



 $\begin{array}{ll} S & \rightarrow NP \ VP \\ S & \rightarrow Aux \ NP \ VP \\ NP & \rightarrow Det \ N \\ NP & \rightarrow Prn \\ NP & \rightarrow NP \ PP \\ VP & \rightarrow V \ NP \\ VP & \rightarrow V \end{array}$

 $\begin{array}{ll} PP & \rightarrow Prp \; NP \\ N & \rightarrow duck \end{array}$

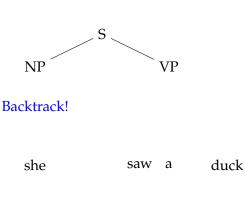
 $egin{array}{ll} N &
ightarrow park \ V &
ightarrow duck \end{array}$

 $VP \rightarrow VP PP$

 $V \rightarrow ducks$

 $V \rightarrow saw$ Prn $\rightarrow she \mid her$

 $\begin{array}{c} \text{Prp} \rightarrow \text{in} \mid \text{with} \\ \text{Det} \rightarrow \text{a} \mid \text{the} \end{array}$



 $\begin{array}{ll} S & \rightarrow NP \ VP \\ S & \rightarrow Aux \ NP \ VP \\ NP & \rightarrow Det \ N \\ NP & \rightarrow Prn \\ NP & \rightarrow NP \ PP \\ VP & \rightarrow V \ NP \\ VP & \rightarrow V \end{array}$

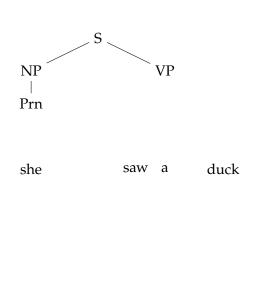
 $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$

 $egin{array}{ll} N &
ightarrow duck \ N &
ightarrow park \ V &
ightarrow duck \ V &
ightarrow ducks \end{array}$

 $V \rightarrow saw$ $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$

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Det \rightarrow a | the

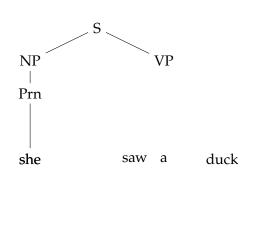


 \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$

 \rightarrow duck $N \rightarrow park$ $V \rightarrow duck$ $V \rightarrow ducks$ \rightarrow saw $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$ Det \rightarrow a | the

N

 $S \rightarrow NP VP$



 $\begin{array}{ll} S & \rightarrow NP \ VP \\ S & \rightarrow Aux \ NP \ VP \\ NP & \rightarrow Det \ N \\ NP & \rightarrow Prn \\ NP & \rightarrow NP \ PP \\ VP & \rightarrow V \ NP \\ VP & \rightarrow V \ PP \end{array}$

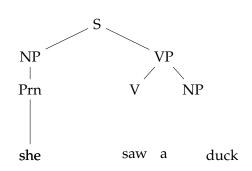
 $\begin{array}{ll} N & \rightarrow duck \\ N & \rightarrow park \\ V & \rightarrow duck \end{array}$

 $PP \rightarrow Prp NP$

 $V \rightarrow ducks$

 $V \rightarrow saw$ Prn $\rightarrow she \mid her$

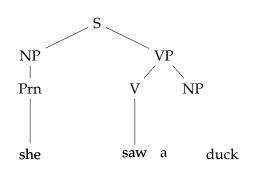
 $Prp \rightarrow sne \mid ner$ $Prp \rightarrow in \mid with$ $Det \rightarrow a \mid the$



 \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ N \rightarrow duck $N \rightarrow park$ $V \rightarrow duck$ $V \rightarrow ducks$

 $V \rightarrow saw$ $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$

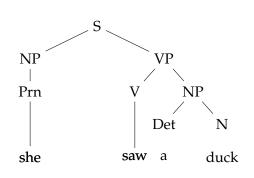
 \rightarrow NP VP



 $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ N \rightarrow duck $N \rightarrow park$ \rightarrow duck $V \rightarrow ducks$ \rightarrow saw

 $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$ $Det \rightarrow a \mid the$

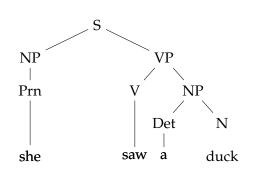
 \rightarrow NP VP \rightarrow Aux NP VP



 \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ N \rightarrow duck $N \rightarrow park$ \rightarrow duck \rightarrow ducks \rightarrow saw

 $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$ $Det \rightarrow a \mid the$

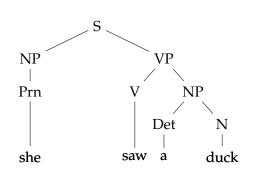
 \rightarrow NP VP



 \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ \rightarrow duck $N \rightarrow park$ \rightarrow duck \rightarrow ducks \rightarrow saw $Prn \rightarrow she \mid her$

 $\operatorname{Prp} \to \operatorname{in} \mid \operatorname{with}$ $\operatorname{Det} \to \operatorname{a} \mid \operatorname{the}$

 \rightarrow NP VP



 \rightarrow NP VP \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ Ν \rightarrow duck $N \rightarrow park$ \rightarrow duck \rightarrow ducks \rightarrow saw $Prn \rightarrow she \mid her$

 $\operatorname{Prp} \to \operatorname{in} \mid \operatorname{with}$ $\operatorname{Det} \to \operatorname{a} \mid \operatorname{the}$

 $S \longrightarrow NP VP$ \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ $N \rightarrow duck$ $N \rightarrow park$ $V \rightarrow duck$ $V \rightarrow ducks$ \rightarrow saw

 $Prn \rightarrow she \mid her$

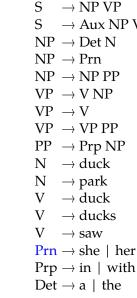
 $Prp \rightarrow in \mid with$

saw

she

a

duck



duck

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a

saw

Prn

she

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 \rightarrow saw

 \rightarrow Aux NP VP



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saw

a

Prn

she

 $S \rightarrow NP VP$

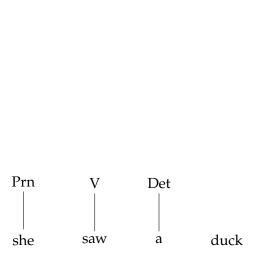
 $NP \rightarrow Det \ N$ $NP \rightarrow Prn$ $NP \rightarrow NP \ PP$ $VP \rightarrow V \ NP$ $VP \rightarrow V$ $VP \rightarrow PP$ $PP \rightarrow Prp \ NP$ $N \rightarrow duck$ $N \rightarrow park$ $V \rightarrow duck$

 $V \rightarrow ducks$ $V \rightarrow saw$ $Prn \rightarrow she \mid her$

 $Prp \rightarrow in \mid with$ $Det \rightarrow a \mid the$

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 \rightarrow Aux NP VP



 $\begin{array}{ll} S & \rightarrow NP \ VP \\ S & \rightarrow Aux \ NP \ VP \\ NP & \rightarrow Det \ N \\ NP & \rightarrow Prn \\ NP & \rightarrow NP \ PP \end{array}$

 $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$

 $VP \rightarrow VPPP$ $PP \rightarrow Prp NP$

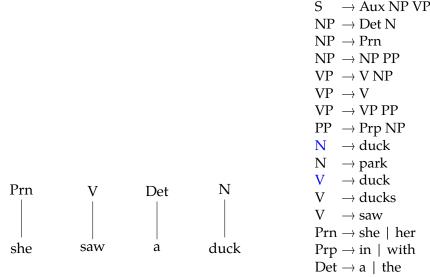
 $\begin{array}{cc}
N & \rightarrow duck \\
N & \rightarrow park
\end{array}$

 $V \rightarrow \text{park}$ $V \rightarrow \text{duck}$

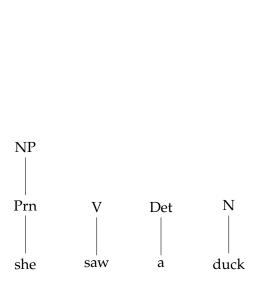
 $V \rightarrow ducks$

 $V \rightarrow saw$ Prn $\rightarrow she \mid her$

 $Prp \rightarrow in \mid with$ $Det \rightarrow a \mid the$



 $S \rightarrow NP VP$

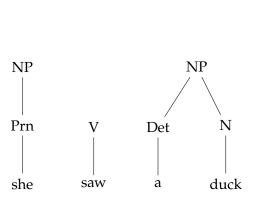


 \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ $N \rightarrow duck$ $N \rightarrow park$ $V \rightarrow duck$ $V \rightarrow ducks$ \rightarrow saw $Prn \rightarrow she \mid her$

 $\operatorname{Prp} \to \operatorname{in} \mid \operatorname{with}$ $\operatorname{Det} \to \operatorname{a} \mid \operatorname{the}$

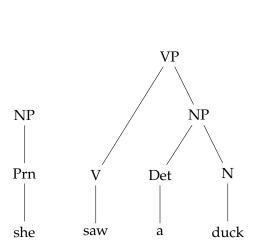
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 $S \rightarrow NP VP$



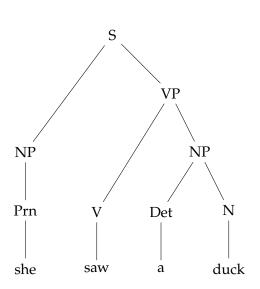
 $S \rightarrow NP VP$ \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ \rightarrow duck $N \rightarrow park$ $V \rightarrow duck$ $V \rightarrow ducks$ \rightarrow saw $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$

Det \rightarrow a | the



 $S \rightarrow NP VP$ \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ \rightarrow duck $N \rightarrow park$ $V \rightarrow duck$ $V \rightarrow ducks$ \rightarrow saw $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$

Det \rightarrow a | the



 $S \rightarrow NP VP$ \rightarrow Aux NP VP $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ $N \rightarrow duck$ $N \rightarrow park$ $V \rightarrow duck$ $V \rightarrow ducks$ \rightarrow saw $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$

Det \rightarrow a | the

Problems with search procedures

- Top-down search considers productions incompatible with the input, and cannot handle left recursion
- Bottom-up search considers non-terminals that would never lead to S
- Repeated work because of backtracking
- ightarrow The result is exponential time complexity in the length of the sentence

Some of these problems can be solved using *dynamic programming*.

CKY algorithm

- The CKY (Cocke–Kasami–Younger) parsing algorithm is a dynamic programming algorithm (Kasami 1965; Younger 1967; Cocke and Schwartz 1970)
- It processes the input *bottom up*, and saves the intermediate results on a *chart*
- Time complexity for *recognition* is $O(n^3)$
- Space complexity is O(n²)
- It requires the CFG to be in *Chomsky normal form* (CNF)

Chomsky normal form (CNF)

- A CFG is in CNF, if the rewrite rules are in one of the following forms
 - $-A \rightarrow BC$
 - $-A \rightarrow a$

where A, B, C are non-terminals and a is a terminal

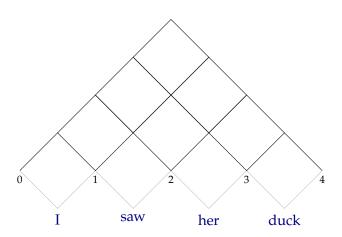
- Any CFG can be converted to CNF
- Resulting grammar is *weakly equivalent* to the original grammar:
 - it generates/accepts the same language
 - but the derivations are different

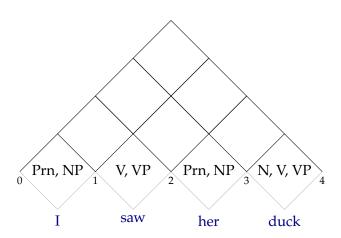
Converting to CNF: example

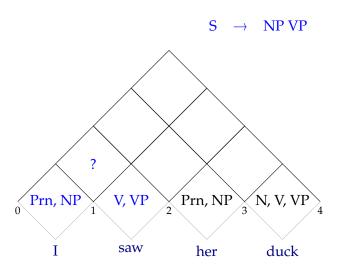
- For rules with > 2 RHS symbols $S \rightarrow Aux NP VP \Rightarrow S \rightarrow Aux X X \rightarrow NP VP$
- For rules with < 2 RHS symbols NP \rightarrow Prn \Rightarrow NP \rightarrow she | her

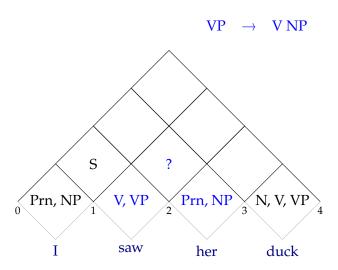
```
S \longrightarrow NP VP
S \rightarrow Aux NP VP
NP \rightarrow Det N
NP \rightarrow Prn
NP \rightarrow NP PP
VP \rightarrow V NP
VP \rightarrow V
VP \rightarrow VP PP
PP \rightarrow Prp NP
N \rightarrow duck
N \rightarrow park
V \rightarrow duck
```

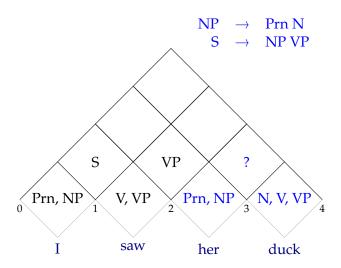
 $V \rightarrow ducks$ $V \rightarrow saw$ $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$ $Det \rightarrow a \mid the$

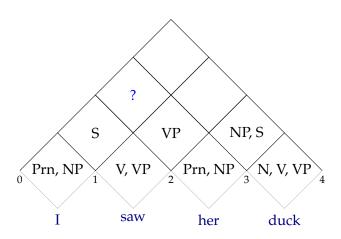


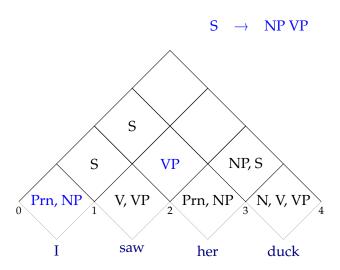


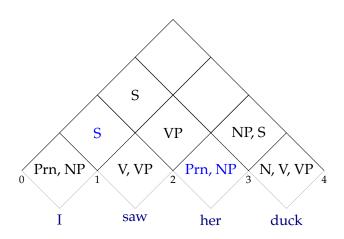


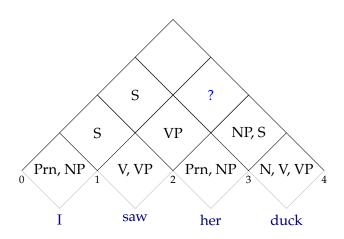


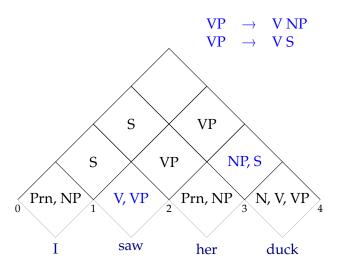


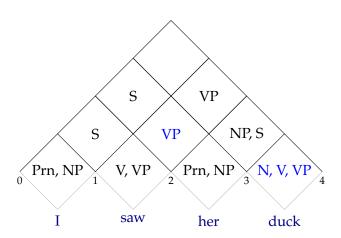


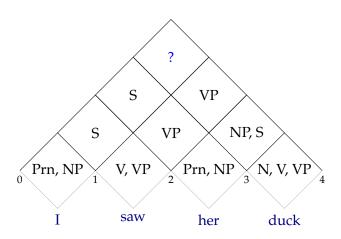


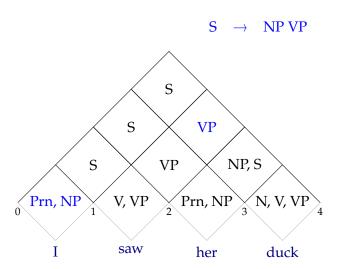


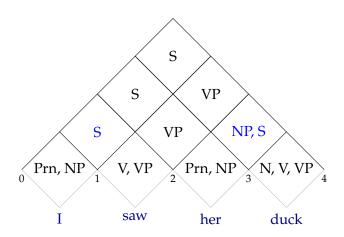


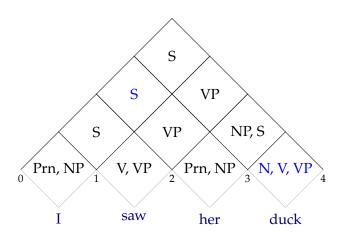












CKY demonstration: the chart

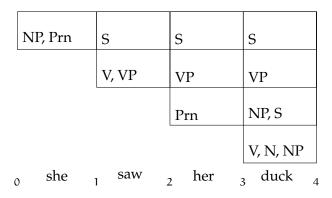


Chart is a 2-dimensional array: $O(n^2)$ space complexity.

Parsing vs. recognition

- We went through a recognition example
- Recognition accepts or rejects a sentence based on a grammar
- For parsing, we want to know the derivations that yielded a correct parse
- To recover parse trees, we
 - we follow the same procedure as recognition
 - add back links to keep track of the derivations

Chart parsing example (CKY parsing)

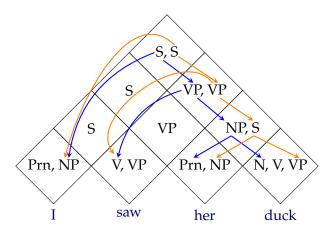
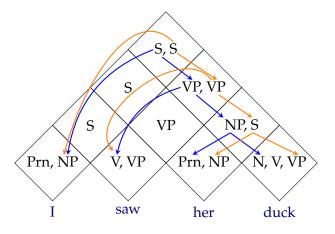


Chart parsing example (CKY parsing)



The chart stores a parse forest efficiently.

CKY summary

- + CKY avoids re-computing the analyses by storing the earlier analyses (of sub-spans) in a table
- It still computes lower level constituents that are not allowed by the grammar
- CKY requires the grammar to be in CNF
- CKY has $O(n^3)$ recognition complexity
- For parsing we need to keep track of backlinks
- CKY can efficiently store all possible parses in a chart
- Enumerating all possible parses have exponential complexity (worst case)

Earley algorithm

- Earley algorithm is a top down (and left-to-right) parsing algorithm (Earley 1970)
- It allows arbitrary CFGs
- Keeps record of constituents that are predicted using the grammar (top-down) in-progress with partial evidence completed based on input seen so far at every position in the input string
- Time complexity is $O(n^3)$

Earley chart entries (states or items)

Earley chart entries are CF rules with a 'dot' on the RHS representing the state of the rule

- A \rightarrow • $\alpha[i,i]$ predicted without any evidence (yet)
- $A \rightarrow \alpha \bullet \beta[i,j]$ partially matched
- $A \to \alpha\beta$ [i,j] completed, the non-terminal A is found in the given span

Earley algorithm: an informal sketch

- 1. Start at position 0, predict S
- 2. Predict all possible states (rules that apply)
- 3. Read a word
- 4. Update the table, advance the dot if possible
- 5. Go to step 2
- 6. If we have a completed S production at the end of the input, the input it recognized

Earley algorithm: three operations

- Predictor adds all rules that are possible at the given state

 Completer adds states from the earlier chart entries that

 match the completed state to the chart entry being

 processed, and advances their dot
 - Scanner adds a completed state to the next chart entry if the current category is a POS tag, and the word matches

Earley parsing example (chart[0])

0 8	she ₁ saw	₂ a	3 duck 4
state	rule	position	operation
0	$\gamma \to \bullet S$	[0,0]	initialization
1	$S \rightarrow \bullet NP VP$	[0,0]	predictor
2	$S \rightarrow \bullet Aux NP VP$	[0,0]	predictor
3	$NP \rightarrow \bullet Det N$	[0,0]	predictor
4	$NP \rightarrow \bullet NP PP$	[0,0]	predictor
5	$NP \rightarrow \bullet Prn$	[0,0]	predictor

Note: the chart[0] is independent of the input.

 \rightarrow NP VP $S \rightarrow Aux NP VP$ $NP \rightarrow Det N$ $NP \rightarrow Prn$ $NP \rightarrow NP PP$ $VP \rightarrow V NP$ $VP \rightarrow V$ $VP \rightarrow VP PP$ $PP \rightarrow Prp NP$ \rightarrow duck $N \rightarrow park$ \rightarrow duck $V \rightarrow ducks$ \rightarrow saw $Prn \rightarrow she \mid her$ $Prp \rightarrow in \mid with$ Det \rightarrow a | the $Aux \rightarrow does \mid has$

Earley parsing example (chart[1])

0	she	1	saw	2 a	₃ duck ₄
sta	ite i	rule		positio	on operation
6]	$Prn \rightarrow$	she •	[0,1]	scanner
7	I	$NP \rightarrow 1$	Prn •	[0,1]	completer
8	S	$S \rightarrow NI$	P •VP	[0,1]	completer
9	I	$NP \rightarrow]$	NP •PP	[0,1]	completer
10	7	$VP \rightarrow \mathbf{Q}$	V NP	[1,1]	predictor
11	7	$VP \rightarrow \mathbf{Q}$	V	[1,1]	predictor
12	7	$VP \rightarrow \P$	VP PP	[1,1]	predictor
13]	$\mathrm{PP} o lacksquare$	Prp NP	[1,1]	predictor

Earley parsing example (chart[2])

0	she	1	saw	2	a	3	duck	4
sta	te ru	le		p	osition	. 0	peratio	n
14	V	→sa	aw •	[]	1,2]	S	canner	
15	V]	${ m P} ightarrow$	V •NP	[]	l <i>,</i> 2]	C	omplete	er
16	V]	${ m P} ightarrow$	V •	[]	l <i>,</i> 2]	C	omplete	er
17	N.	$\mathrm{P} \rightarrow$	•Det N	[2	2,2]	p	redicto	r
18	N.	$\mathrm{P} \rightarrow$	•NP PP	[2	2,2]	p	redicto	r
19			∙Prn	[2	2,2]	p	redicto	r
20	S-	→N	P VP •	[(),2]	p	redicto	r

Earley parsing example (chart[3])

0	she	1	saw	2	a	3	duck	4
	state	rule		po	sition	0]	peration	
_	21 22	Det − NP →	a • Det •N	[2, [2,	-		canner ompleter	r

Earley parsing example (chart[4])

_o sh	e ₁ saw	2 a	3 duck 4
state	rule	position	operation
23	$N \rightarrow duck \bullet$	[3,4]	scanner
24	$V \rightarrow duck \bullet$	[3,4]	scanner
25	$NP \rightarrow Det N \bullet$	[2,4]	completer
26	$VP \rightarrow V NP \bullet$	[1,4]	completer
27	$S \rightarrow NP VP \bullet$	[0,4]	completer

Earley parsing: summary

- Top-down approach with bottom-up filtering (better filtering may be achived with lookahead)
- It can process any CFG (no need for CNF)
- Complexity is the same as CKY
 - time complexity : $O(n^3)$
 - space complexity: $O(n^2)$
- Our examples show recognition, we need to maintain backlinks for parsing
- Again, Earley chart stores a parse forest compactly, but extracting all trees may require exponential time

An exercise

Construct the CKY and Earley charts for the following sentence

The duck she saw is in the park

Recommended grammar:

```
S \rightarrow NP VP
                              PP \rightarrow Prp NP
                              N \rightarrow park
NP \rightarrow Det N
                              N \rightarrow duck
NP \rightarrow Prn
                              V \rightarrow duck
NP \rightarrow NP PP
NP \rightarrow NP S
                              V \rightarrow saw
VP \rightarrow V NP
                              Prn \rightarrow she
VP \rightarrow V
                              Prp \rightarrow in
                              Det \rightarrow the
VP \rightarrow VP PP
```

Summary: context-free parsing algorithms

- Naive search for parsing is intractable
- Dynamic programming algorithms allow polynomial time recognition
- Parsing may still be exponential in the worse case
- Charts represent ambiguity, but cannot say anything about which parse is the best

Pretty little girl's school

Natural languages and ambiguity



Cartoon Theories of Linguistics, SpecGram Vol CLIII, No 4, 2008. http://specgram.com/CLIII.4/school.gif

Some more examples

- Lexical ambiguity
 - She is looking for a match
 - We saw her duck
- Attachment ambiguity
 - I saw the man with a telescope
 - Panda eats hamboo shoots and leaves
- Local ambiguity (garden path sentences)
 - The horse raced past the barn fell
 - The old man the boats
 - Fat people eat accumulates

We use statistical methods for dealing with ambiguity (not in this course).

References / additional reading material

• Jurafsky and Martin (2009, Chapter 13)

References / additional reading material (cont.)

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